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BEHAVIOUR OF PRETENSIONED REINFORCED
RECTANGULAR CONCRETE BEAMS SUBJECTED
TO COMBINED LOADING

by

K.L. SORENSEN



A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

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APRIL, 1968



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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Behaviour of Pretensioned Reinforced Rectangular Concrete Beams Subjected to Combined Loading", submitted by K.L. Sorensen in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

This report is based on the results of Part I of a Four-Part behavioural study being carried out at the Structural Laboratory of the University of Alberta by Dr. P. Mukherjee (*) under the guidance of Dr. J. Warwaruk (+). The results of the complete study will be reported at a later date.

This section of the study was designed to gain further knowledge of the behaviour of pretensioned reinforced rectangular concrete beams subjected to torsion and flexure.

Twelve beams having a nominal 6 x 12 inch cross-section were tested. All beams contained the same amount of mild steel reinforcement. Six beams were concentrically prestressed and six were eccentrically prestressed with an eccentricity ratio of 0.167. The ratio of torsional moment to flexural moment was varied for each type of prestressing.

The test results are presented in the form of tables, graphs, and interaction diagrams in Appendix A.

(*) Dr. P.R. Mukherjee
Post-Doctoral Research Associate
University of Alberta

(+) Dr. J. Warwaruk
Associate Professor
Dept. of Civil Engineering
University of Alberta
Edmonton, Alberta

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CHAPTER I

INTRODUCTION

1.1 Introductory Remarks

Only nominal provisions are presently available for torsion. The interaction of slabs monolithic with spandrel beams has long been the classic example of an area where torsion has existed. However, the conservative approach to other effects such as flexure and shear has always left torsion as a very minor effect which could, in most cases, be ignored. This is not the case with present-day design. The continual change in analysis and design methods, design load and safety factors, along with a better understanding of the total behaviour of a structure, is placing an increasing amount of importance on the effects of torsion.

A vast amount of research has been carried out to investigate the behaviour of concrete members when subjected to flexure, shear, pre-stress, and varying combinations of these effects. The study of concrete members subjected to torsion is a relatively new field but one which is an obvious step in the progress of research. Eventually the behaviour of concrete members under the combined action of flexure, shear, pre-stress, and torsion will be understood. This will enable more functional, modern, and aesthetic structures to be designed and constructed within the realm of practicality.

1.2 OBJECT

The primary object of this investigation was to study the behaviour of reinforced prestressed rectangular concrete beams subjected to torsion and flexure.

The variables in this study were:

(a) the type of prestress

(i) concentric

(ii) eccentric

(b) varying torsional moment to bending moment (ϕ) ratios.

All strands were to be stressed to the same level of prestress however, some slight variations did occur and these are explained where applicable.

The effects of elastic shortening upon release of the prestressing strands, and the time dependent strains, from the time of release until the time of testing, were measured. The behaviour of both the longitudinal and transverse reinforcement was measured by strain gauges located at appropriate positions.

The beams were all manufactured and tested according to the procedures in Chapter III. The results of the tests are presented in the form of various tables, diagrams, and comments.

1.3 SCOPE

The investigation included tests of two series of beams. The first series consisted of beams 101 to 106 and the second, beams 121 to

126. The rectangular cross-section of all beams measured 6 x 12 inches and had an effective depth of 11 inches. All beams, 10 ft. 6 in. in length, were prestressed using high strength steel strand. Table 3.3 presents the physical properties of all twelve specimens.

The test equipment, as outlined in Chapter III, allowed the specimens to be tested in pure torsion, pure flexure, or a varying combination of these two effects. The results of the tests are presented as MOMENT-DEFLECTION curves, TORQUE-TWIST curves, and DIMENSIONAL and NON-DIMENSIONAL INTERACTION DIAGRAMS, as well as in tabular and discussion form. This form of presentation could be useful to other investigators.

CHAPTER II

REVIEW OF RESEARCH

2.1 INTRODUCTION

The amount of research to date on the behaviour of prestressed concrete beams subjected to combined loading has been limited. The articles most pertinent to this investigation are summarized in this chapter.

2.2 PREVIOUS RESEARCH

2.2.1 NYLANDER

In 1945, Nylander (1) carried out tests on longitudinally reinforced prismatic beams subjected to torsion and bending. The specimens he tested did not contain any special reinforcement to resist torsional or flexural shearing stresses.

The general conclusions of Nylander's study were:

(1) The presence of axial compression or prestress tends to flatten out the angle of crack formation due to a modification of the principal stresses.

(2) The presence of an applied bending moment, not exceeding the flexural cracking moment, was beneficial to the torsional resistance of the specimen.

(3) A small amount of torque, not exceeding the torsional cracking moment, increased the bending moment capacity when failure was primarily due to flexure.

In general, Nylander considered the validity of various theories for predicting the ultimate strength of members subjected to prestress and torsion but he chose to accept the plastic theory approach provided the effect of prestress was taken into account when calculating the tensile strength of the concrete.

2.2.2 COWAN and ARMSTRONG

In 1955, Cowan and Armstrong (2) conducted tests to obtain information on the behaviour of reinforced and prestressed concrete beams subjected to combined bending and torsion.

They concluded that:

(1) The crack formation was dependent on the ratio of bending to twisting. Flexural cracks had a vertical orientation while pure torsional cracks were oriented at approximately 45° to the longitudinal axis of the beam. The orientation of cracks for torsional to flexural moment ratios between 0 and ∞ ranged between the limits mentioned above. Further, the addition of prestress flattened the crack orientation to possible angles of less than 45° , depending upon the level of prestress.

(2) The specimens exhibited a great reduction in stiffness once they had cracked but they did possess additional capacity over and above their cracking capacity. This additional capacity was attributed to the reinforcement and was dependent not only on the amount of reinforcement but also on its location within the section. Transverse reinforcement was not included in their test specimens and was therefore not discussed.

(3) The presence of a bending moment, less than the flexural cracking moment, was beneficial to the torsional capacity of a specimen and the presence of a torsional moment, less than the torsional cracking moment, was beneficial to the flexural capacity of the specimen.

Cowan advocated the elastic theory approach. He concluded generally that closer agreement between experimental results and theory would be obtained if the torsional strength of the concrete was considered and the reinforcement required to take only those tensile stresses which exceed the tensile strength of the concrete.

2.2.3 ZIA

In 1961 Zia (3) conducted tests on prestressed rectangular concrete beams containing some mild steel reinforcement. These specimens, subjected to pure torsion, formed a small part of a larger study but only the applicable conclusions are summarized here.

Zia concluded that:

(1) The presence of prestress increased the torsional resistance of the specimens.

(2) The presence of prestress caused the angle of crack formation to be less than that observed in similar unprestressed specimens.

(3) The presence of web reinforcement gave the specimens increased ductility after cracking and an ultimate torque capacity above the cracking torque.

Zia's general recommendation for prestressed concrete specimens with steel reinforcement was to use Cowan's strain energy approach which adds the torsional capacities of both steel and concrete to obtain the ultimate torque capacity of the specimen.

2.2.4 SWAMY

Swamy (4) reported on tests conducted on twenty rectangular hollow prestressed concrete beams with comparison tests on prestressed solid and plain hollow concrete beams. The specimens were subjected to a variety of load combinations ranging from pure bending to pure torsion.

The conclusions of his study were:

(1) The torsional resistance of a hollow beam can be increased by the presence of bending moments.

(2) The mode of failure, either by crushing in compression or by diagonal tension, is dependent on the ratio of torque to bending. The transition stage, from one failure mode to the other, is very complex because of considerable inelastic behaviour prior to a torsion failure.

(3) The presence of a compressive stress (prestress) alters the principal stresses and tends to arrest the development of failure strains thus increasing the capacity of the beam in torsion.

(4) The failure of concrete under combined stresses cannot be adequately explained by any single property of the material. Empirical interaction curves are the best method for assessing the strength under various stress combinations ranging from pure bending to pure torsion.

2.2.5 McMULLEN

The tests conducted by McMullen (5) in 1967 were carried out on reinforced rectangular concrete beams subjected to various degrees of combined load. Since none of these specimens were prestressed no conclusions as to the effect of prestressing are available however the effects of web reinforcement and the torque-to-bending (ϕ) ratio were discussed and are briefly summarized here.

McMullen concluded that:

(1) Prior to cracking the web reinforcement had very little effect on the behaviour of the specimens, however, after cracking occurred the stiffness was dependent on the amount and spacing of the transverse reinforcement. Increasing the spacing of the transverse reinforcement decreased both the slope of the TORQUE-TWIST curve after cracking and the ultimate torsional capacity of the specimen.

(2) The length of the initial straight portion of a TORQUE-TWIST curve was observed to decrease with decreasing values of ϕ thus indicating that the presence of bending moment reduced the torsional moment at which cracking of the concrete section occurred.

2.2.6 MUKHERJEE

In 1967 Mukherjee (6) conducted tests on prestressed reinforced rectangular beams subjected to pure torsion. His pertinent conclusions are summarized here.

He concluded that:

(1) The cracking torque of a prestressed beam containing longitudinal and transverse reinforcement was comparable to that of an equivalent prestressed beam without reinforcement. Therefore, the effect of reinforcement may be ignored when calculating the cracking torque.

(2) The amount of ductility exhibited by the prestressed beams was dependent upon the amount and type of reinforcement.

(3) The minimum amount of transverse reinforcement required to ensure ductility should be related to the level of prestress, the yield strength of the reinforcing steel, the tensile strength of the concrete and the dimensions of the cross-section.

(4) The level of prestress was observed to influence the region of behaviour between the lower limit of cracking torque and the upper limit of compressive failure torque. The extent of this region was observed to decrease as the level of prestress was increased thus reducing both the beams post-cracking capacity for load and the amount of ductility exhibited by the beam. Thus, ductile behaviour can only be insured if the level of prestress is limited.

(5) The ultimate torque was composed of two additive parts. The first part was independent of the prestress and cracking torque, the second part was proportional to the dimensions, yield strength, and spacing of the transverse reinforcement, and the inclination of the crack formation.

Mukherjee generally concluded that much more test data was required in order to formulate a theoretical means of estimating the behaviour of prestressed reinforced concrete beams.

CHAPTER III
MATERIALS, FABRICATION, TEST EQUIPMENT
AND TEST PROCEDURES

3.1 MATERIALS

3.1.1 CONCRETE

The mix design used for the test specimens was:

- | | |
|-----------------------|----------|
| (1) CEMENT (TYPE III) | 133 lbs. |
| (2) SAND | 344 lbs. |
| (3) COARSE AGGREGATE | 500 lbs. |

The amount of water used, per mix, ranged from 83 lbs. to 86 lbs. and averaged 85 lbs. This mix yielded seven cubic feet of concrete with a 3 in. slump.

3.1.2 SAND

A sieve analysis of the sand is given in Table 3.1. The average moisture content was about 4%.

TABLE 3.1
SIEVE ANALYSIS OF SAND

SIEVE SIZE	WEIGHT RETAINED (gms.)	% RETAINED	CUMULATIVE % RETAINED	A.S.T.M. STANDARD
# 4	17.5	3.0	3.0	0 - 5
# 8	85.2	14.7	17.7	
# 16	54.6	9.5	27.2	20 - 55
# 30	60.0	10.3	37.5	
# 50	208.4	35.8	73.3	70 - 90
#100	122.9	21.1	94.4	90 - 98
PAN	17.8	3.1	-	
SILT	14.4	2.5	-	
TOTAL	580.8	100.0	253.1	
FINENESS MODULUSS	2.53			

3.1.3 COARSE AGGREGATE

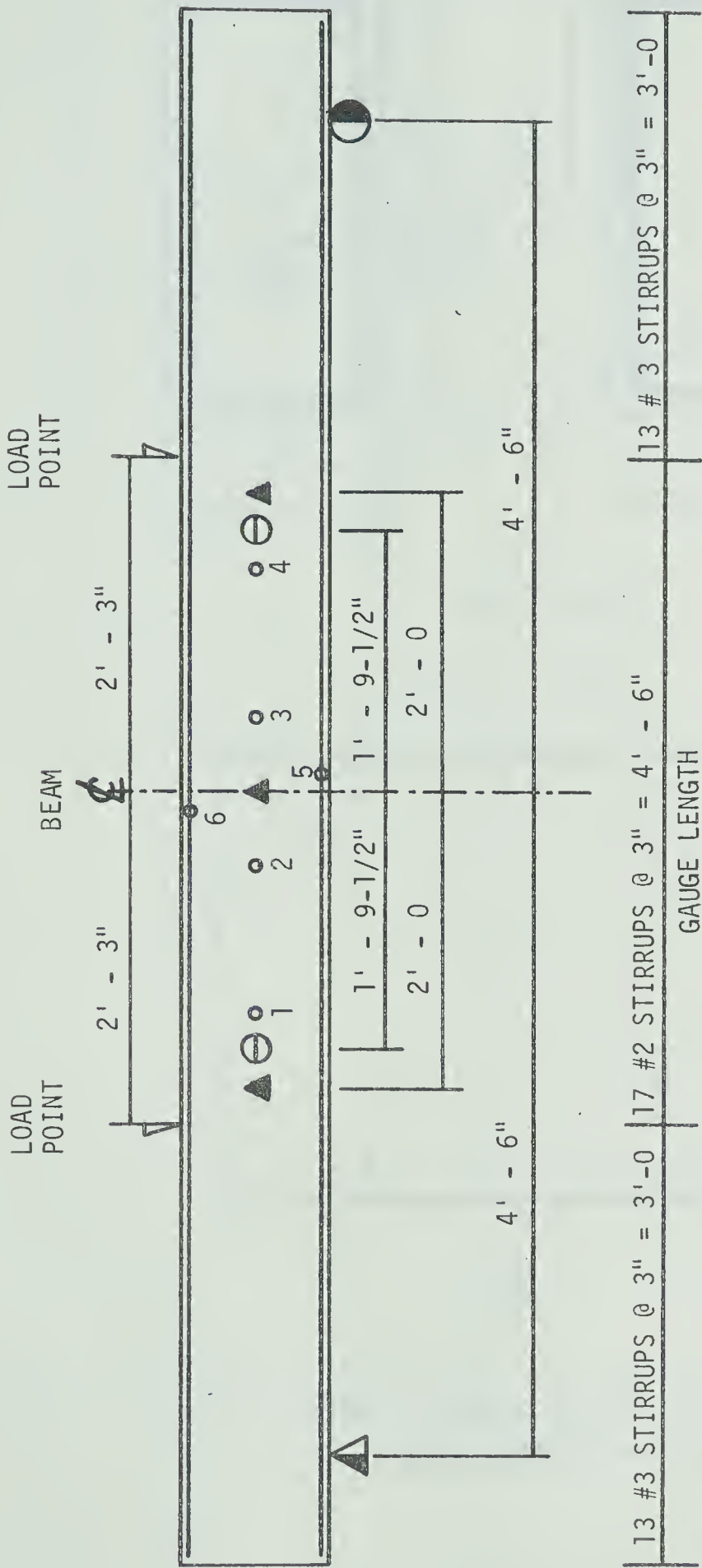
The coarse aggregate was 3/4" maximum size crushed rock with an average moisture content of 1.7%. The results of a sieve analysis are presented in Table 3.2.

TABLE 3.2
SIEVE ANALYSIS OF COARSE AGGREGATE

SIEVE SIZE	WEIGHT RETAINED (lbs.)	% RETAINED	CUMULATIVE % RETAINED
3/4"	0.30	1.1	1.1
3/8"	15.63	58.4	59.5
# 4	10.03	37.5	97.0
PAN	0.80	3.0	100.0
TOTAL	26.76	100.0	257.6

BEAM NO.	LONGITUDINAL REINFORCEMENT			TRANSVERSE REINFORCEMENT			CONCRETE STRENGTH		PRESTRESS	
	DESCRIPTION	P ₁ %	f _{y1} ksi	DESCRIPTION	P _t %	f _{yt} ksi	f' _c psi	f' _{sp} psi	TOTAL EFFECTIVE STRESS	ECCEN- TRICITY RATIO
101	4 # 3	0.61	51.7	#2 @ 3	0.70	45.5	5500	502	33.80	0
102	4 # 3	0.61	51.7	#2 @ 3	0.70	45.5	5000	437	33.20	0
103	4 # 3	0.61	51.7	#2 @ 3	0.70	45.5	4879	490	35.13	0
104	4 # 3	0.61	51.7	#2 @ 3	0.70	45.5	4936	411	31.98	0
105	4 # 3	0.61	51.7	#2 @ 3	0.70	45.5	5202	486	35.31	0
106	4 # 3	0.61	50.9	#2 @ 3	0.70	45.5	5114	498	32.12	0
121	4 # 3	0.61	50.9	#2 @ 3	0.70	45.5	4936	428	35.24	0.167
122	4 # 3	0.61	50.9	#2 @ 3	0.70	45.5	5260	527	35.24	0.167
123	4 # 3	0.61	50.9	#2 @ 3	0.70	45.5	4630	435	36.65	0.167
124	4 # 3	0.61	50.9	#2 @ 3	0.70	45.5	4458	394	36.65	0.167
125	4 # 3	0.61	50.9	#2 @ 3	0.70	45.5	5508	441	36.37	0.167
126	4 # 3	0.61	50.9	#2 @ 3	0.70	45.5	5462	479	36.37	0.167

TABLE 3.3 PROPERTIES OF TEST SPECIMENS



▲ LOCATION OF TWO DEFLECTION GAUGES (ONE EACH SIDE)

⊖ LOCATION OF ROTATION GAUGE

○ STRAIN GAUGE LOCATION (SR-4 ELECTRICAL)

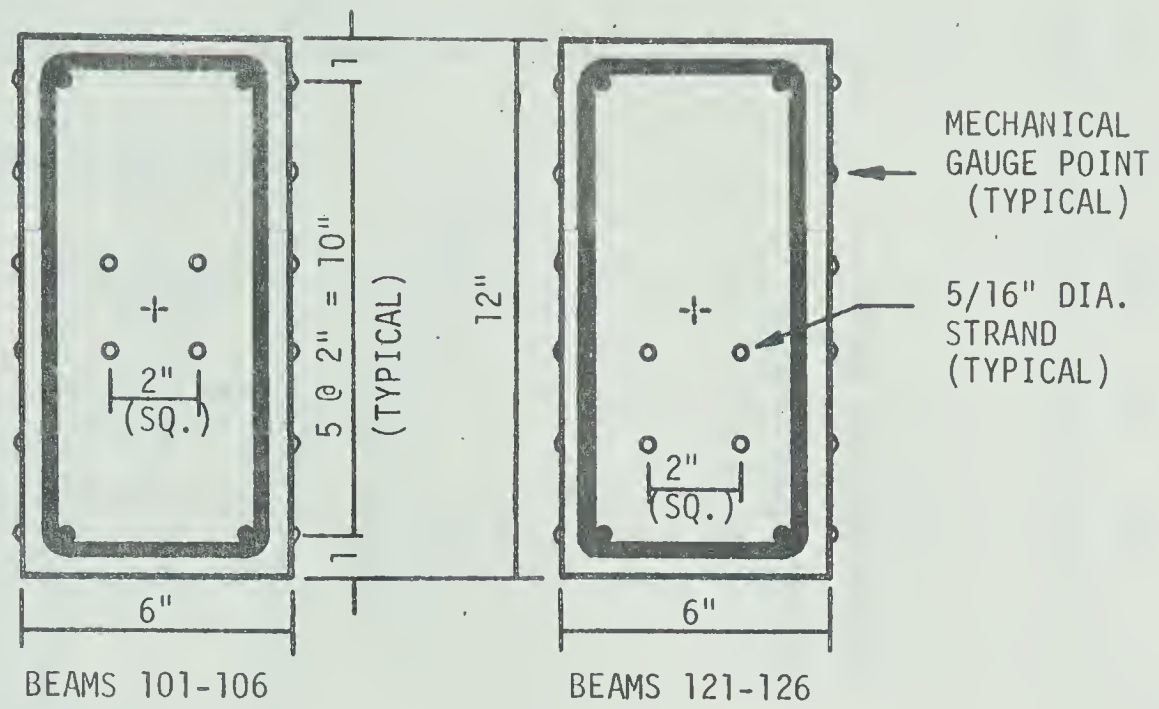
1, 2, 3, & 4 - STIRRUPS

5 & 6 - LONGITUDINAL STEEL
(DIAGONALLY OPPOSED)

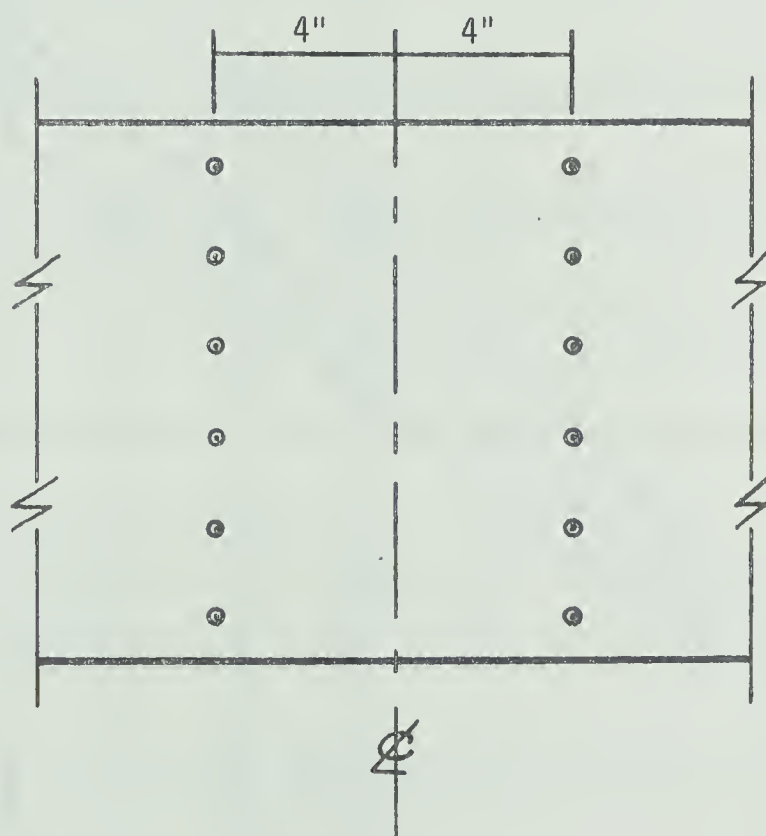
STEEL IN GAUGE LENGTH			
BEAM	LONG.	TRANS.	
101-105	# 3A	# 2	
106	# 3B	# 2	
121-126	# 3B	# 2	

NOTE:- A & B INDICATE DIFFERENT HEATS
- PRESTRESS STRANDS NOT SHOWN

FIGURE 3.1 TYPICAL SPECIMEN

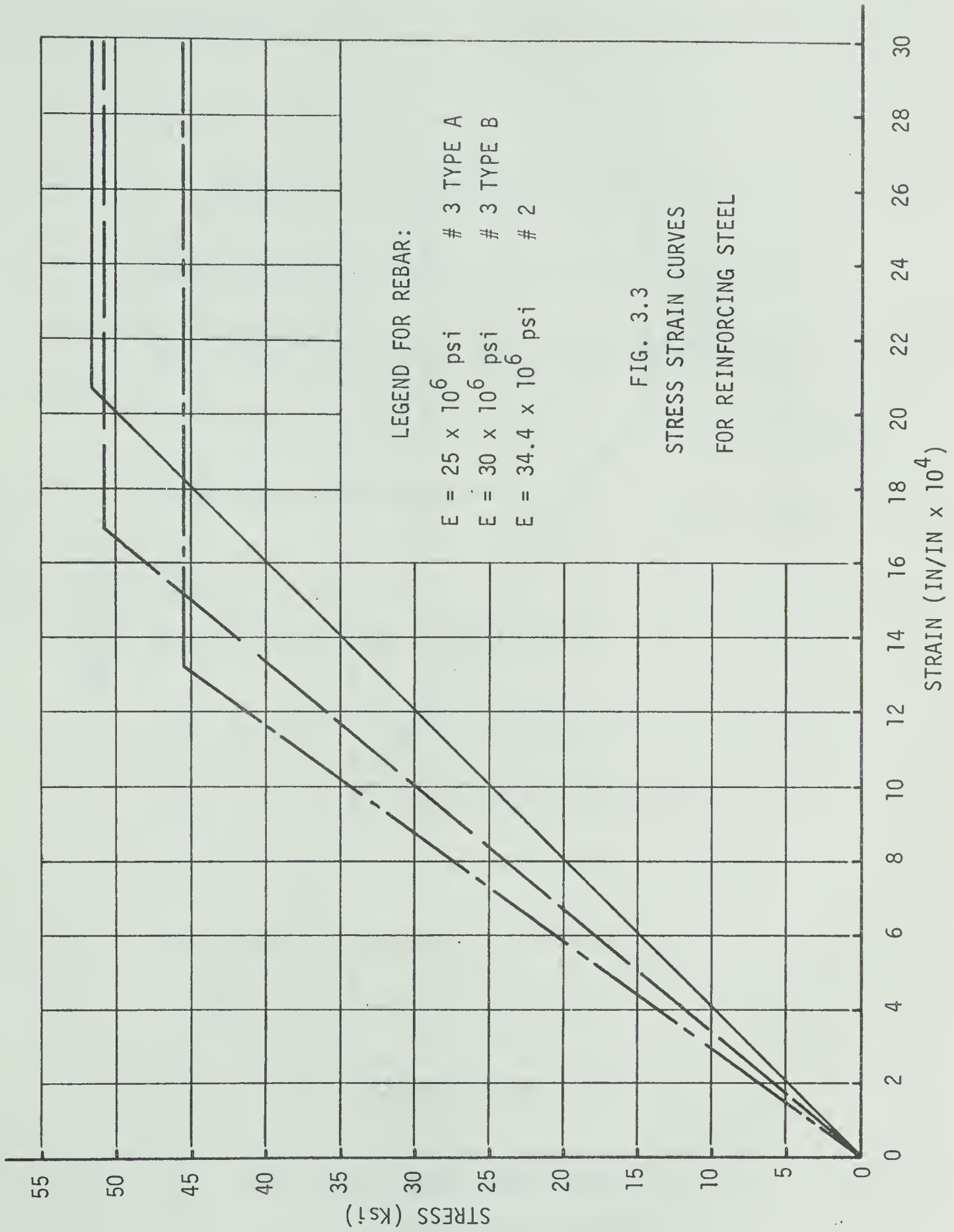


(a) Cross Sections



(b) Side View Showing Mechanical Gauge Points (Opposite Side Same)

FIG. 3.2 BEAM SECTIONS



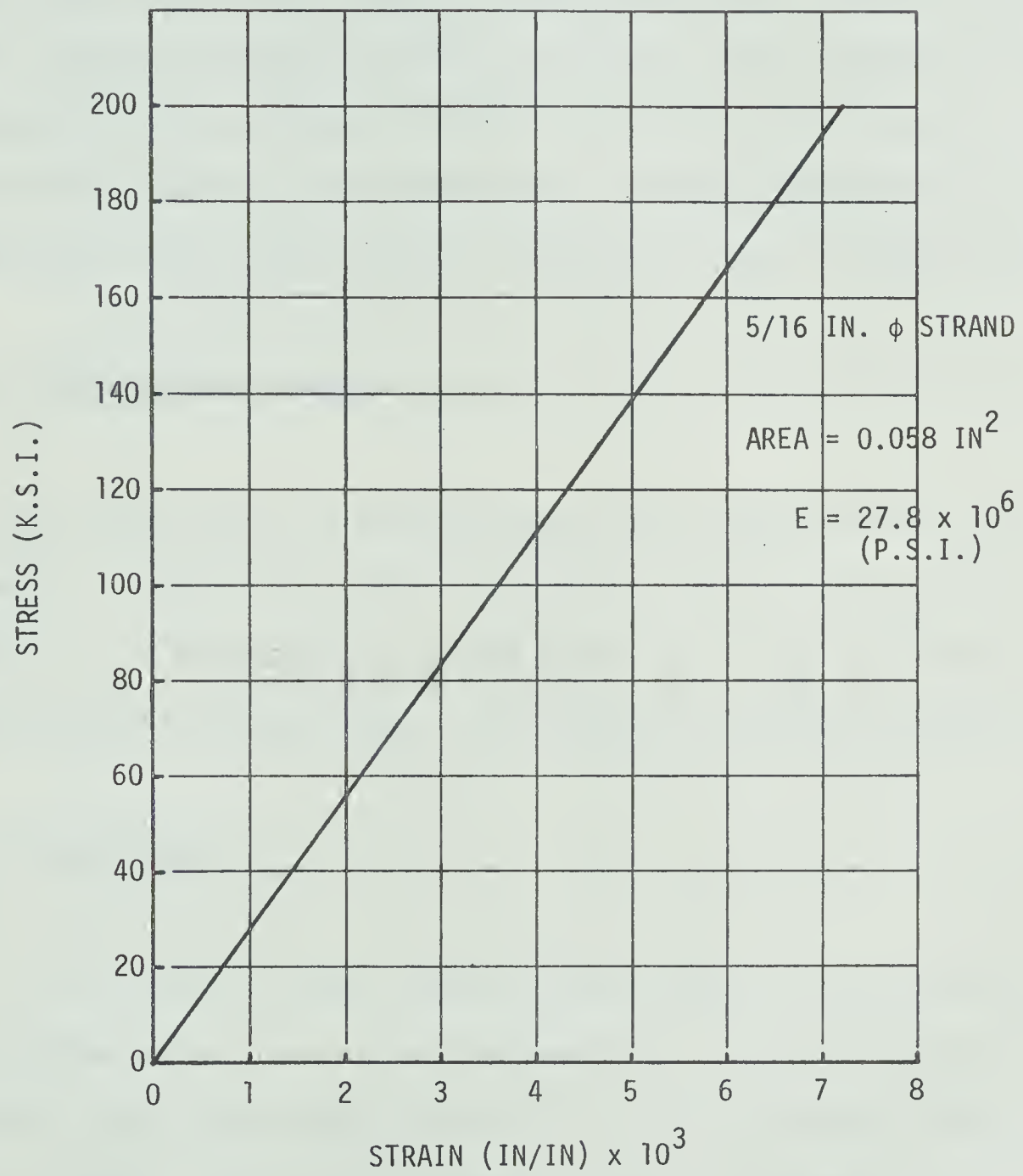


FIG. 3.4 STRESS-STRAIN CURVE
FOR PRESTRESSING STRAND

3.1.4 REINFORCEMENT

The non-prestressed reinforcement used in the test specimens is described in Table 3.3. The # 3 deformed bars were from two different heats and are designated as TYPE A or TYPE B. The arrangement of reinforcement in the specimens is shown in Figure 3.1 and Figure 3.2. Representative samples of each type of # 3 deformed bar and the # 2 smooth bars were tested to obtain the results shown in Figure 3.3.

3.1.5 PRESTRESSING STRAND

The strand used for prestressing the test specimens was 5/16" diameter - 7 wire strand with a guaranteed minimum yield strength of 235 K.s.i. A representative strand sample was tested to obtain its stress-strain characteristics. The results are shown in Figure 3.4.

3.2 FABRICATION

The reinforcing steel cages were fabricated by wiring the ties and longitudinal steel bars at each intersection. At strain gauge locations the bars were ground smooth and Type A-7 SR - 4 electrical strain gauges were attached. These strain gauges were then waterproofed with three coats of GW-2 waterproofing compound and wrapped with tape to ensure their safety during casting. The reinforcing steel cages having been completed, the forms were then cleaned, assembled, placed in position between the concrete bulkheads and oiled. The reinforcing steel cages

were then placed in the forms and properly positioned.

Prior to the placing of the strand, the system in a longitudinal direction, comprised the following:

- (a) end bearing plates bearing on,
- (b) a concrete bulkhead fastened to the laboratory floor by four prestressed bolts,
- (c) two, three-channel forms in series, including end form plates and reinforcing cages,
- (d) the opposing concrete bulkhead with,
- (e) a set of end bearing plates.

Once the system was properly aligned the prestressing strands were threaded through the system. The strand length was such that the strand extended four to five feet past the end of each bulkhead. At the north bulkhead, load cells and wedge-grip end anchorage units were installed. A typical arrangement is shown in Figure 3.5. The south bulkhead was used as the stressing point and wedge-grip end anchorages were used to lock the strand once stressing was complete.

The strands were individually stressed using a Simplex center-hole hydraulic jack operated by a Blackhawk hand pump. The jacking arrangement is illustrated by Figure 3.6. The original intent was to prestress each strand to an equal level of prestress. This was not achieved but the differences in stress level were minor in any one beam.

The concrete was mixed in a nine cubic foot capacity laboratory mixer. One batch was sufficient for each beam including its control cylinders. Prior to mixing, a butter mix was used to condition the mixer.

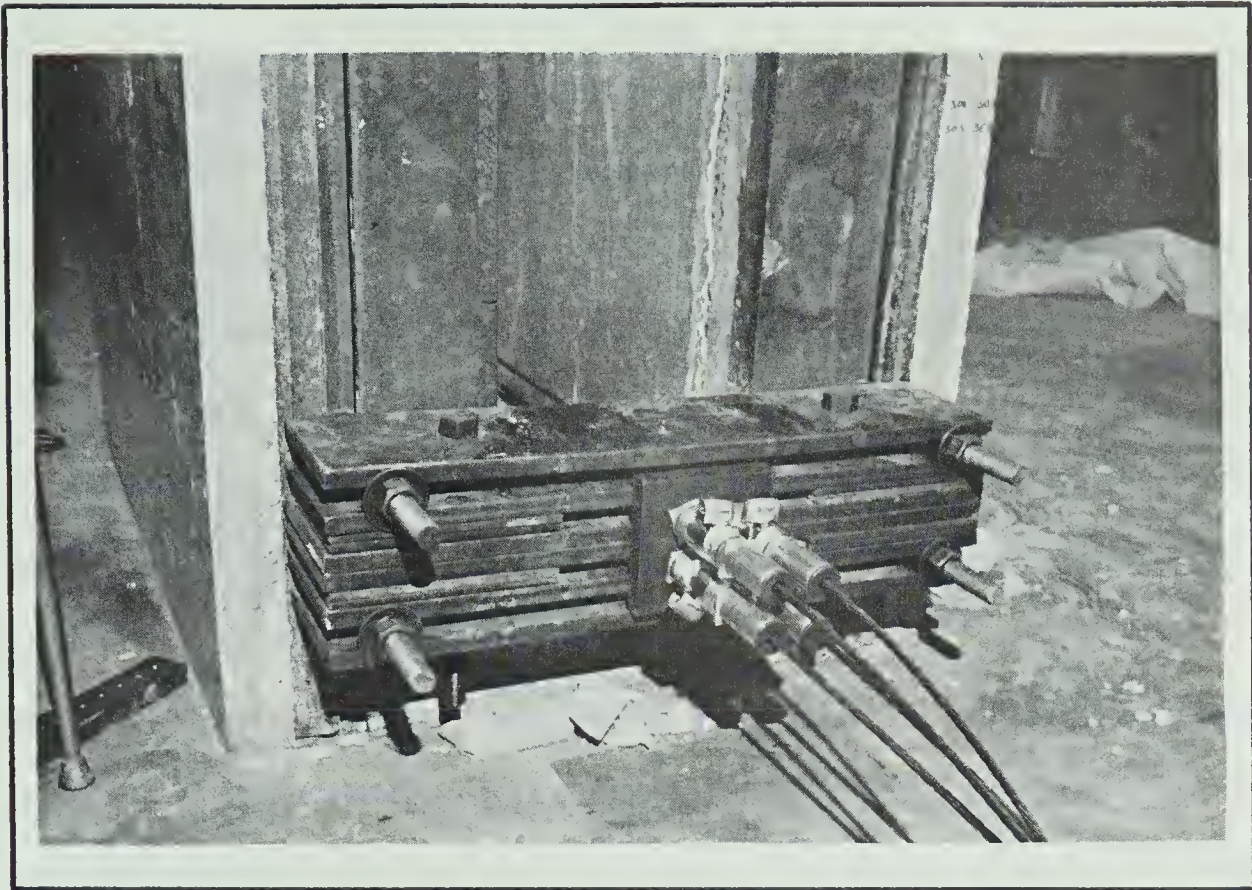


FIGURE 3.5 NORTH BULKHEAD

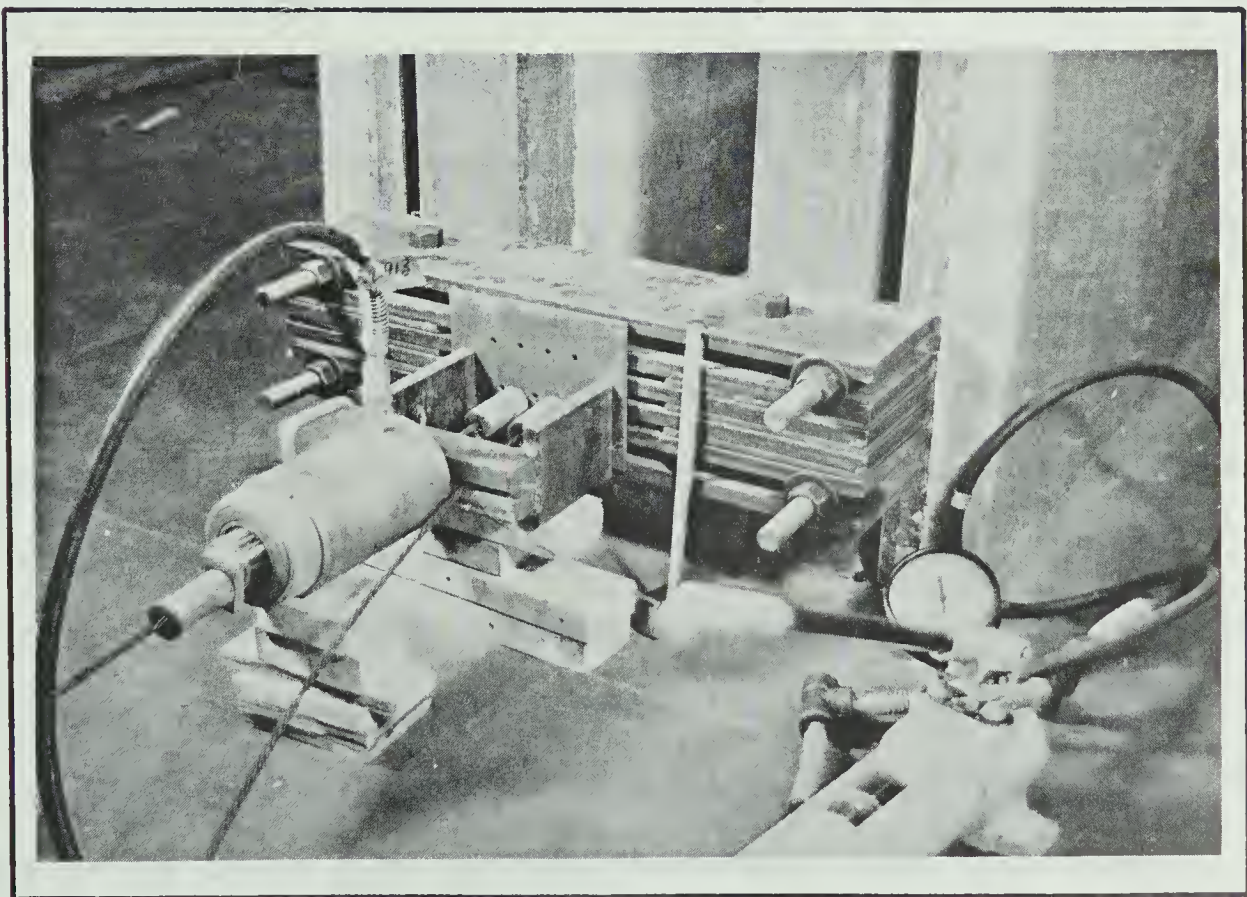


FIGURE 3.6 SOUTH BULKHEAD
(JACKING SYSTEM)

The concrete was mixed for approximately five minutes. The water content was adjusted until a 3" slump was obtained. The concrete was then deposited in the forms and vibrated with a high frequency internal vibrator.

Five six by twelve inch concrete cylinders were cast with each beam. The cylinders were cured and stored under the same conditions as the beam. Three cylinders were used for compression tests while the remaining two were subjected to the tensile splitting test. All cylinders were tested on the same day as the corresponding beam test.

The beams and cylinders were cured for twenty four hours by covering them with wet burlap and a plastic sheet. The day after casting the forms were stripped and the wet burlap and plastic replaced, allowing a further six days of moist curing.

At seven days the burlap and plastic were removed and final readings taken on each load cell on each strand. This eliminated any error in strand forces due to relaxation of the steel strands.

After the beams had been exposed to the air for about four hours the sides and tops were relatively dry. Mechanical gauge points were positioned on the specimens and the initial readings taken with an 8 in. DEMEC deformation gauge. The location of these gauge points is illustrated in Figures 3.1 and 3.2. After these readings were taken and recorded, the strands were then subjected to heat from a cutting torch. This effected a gradual transfer of stress from the strands to the concrete.

Immediately after release, the mechanical gauge points were again read and these readings recorded. The change in readings resulting from release of the strand gave the elastic shortening due to prestress.

The beams along with their control cylinders were then set aside for air storage until the time of the test.

3.3 TEST EQUIPMENT

The equipment used to apply the bending moment and the equipment used to apply the twisting moment were two independent systems. A 100 kip Amsler jack was used to apply transverse load to a distributing beam that was supported at one end by a ball and at the other end by a roller. The ball and the roller each rested on a roller pad assembly which in turn rested on a pipe collar. These pipe collars were clamped to the specimen and allowed the specimen to twist freely.

The east end of the specimen was supported by the twisting head which is illustrated by Figure 3.7. The torsional loading equipment was located beneath the laboratory floor directly under the twisting head. The torsional loading equipment is illustrated by Figure 3.8. The west end of the specimen was supported by the fixed head which is illustrated by Figure 3.9. A complete equipment arrangement for combined loading is shown in Figure 3.10.

The force in the torsional loading cables was controlled by means of a hand operated hydraulic pump and a Baldwin Lima Hamilton strain indicator. The SR-4 electrical strain gauge readings were measured with a Budd strain indicator.

A complete detailed discussion of this test equipment was given by McMullen (5) and information presented here is designed only as a brief resumé for purposes of this thesis.

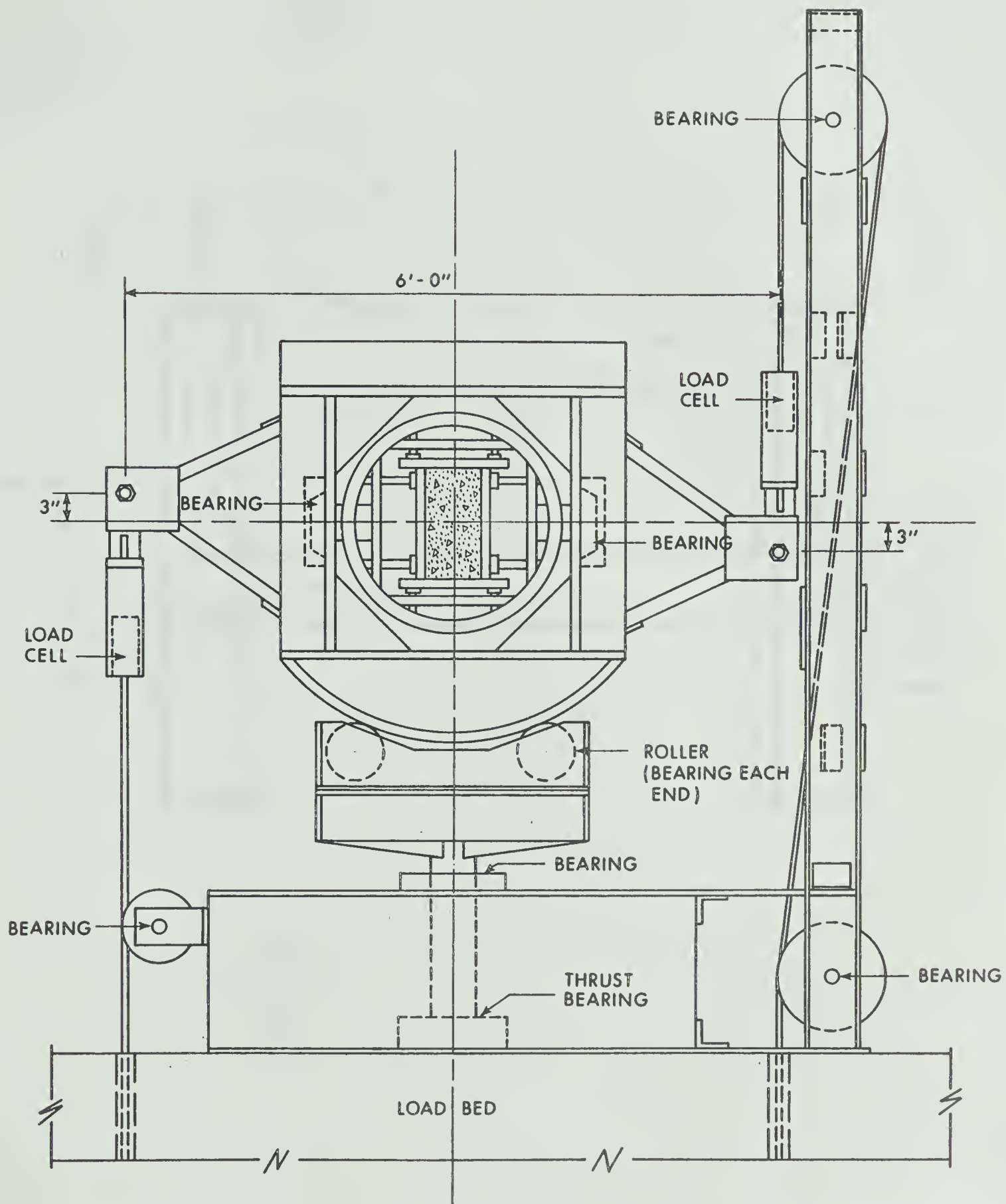


FIG. 3.7 TWISTING HEAD

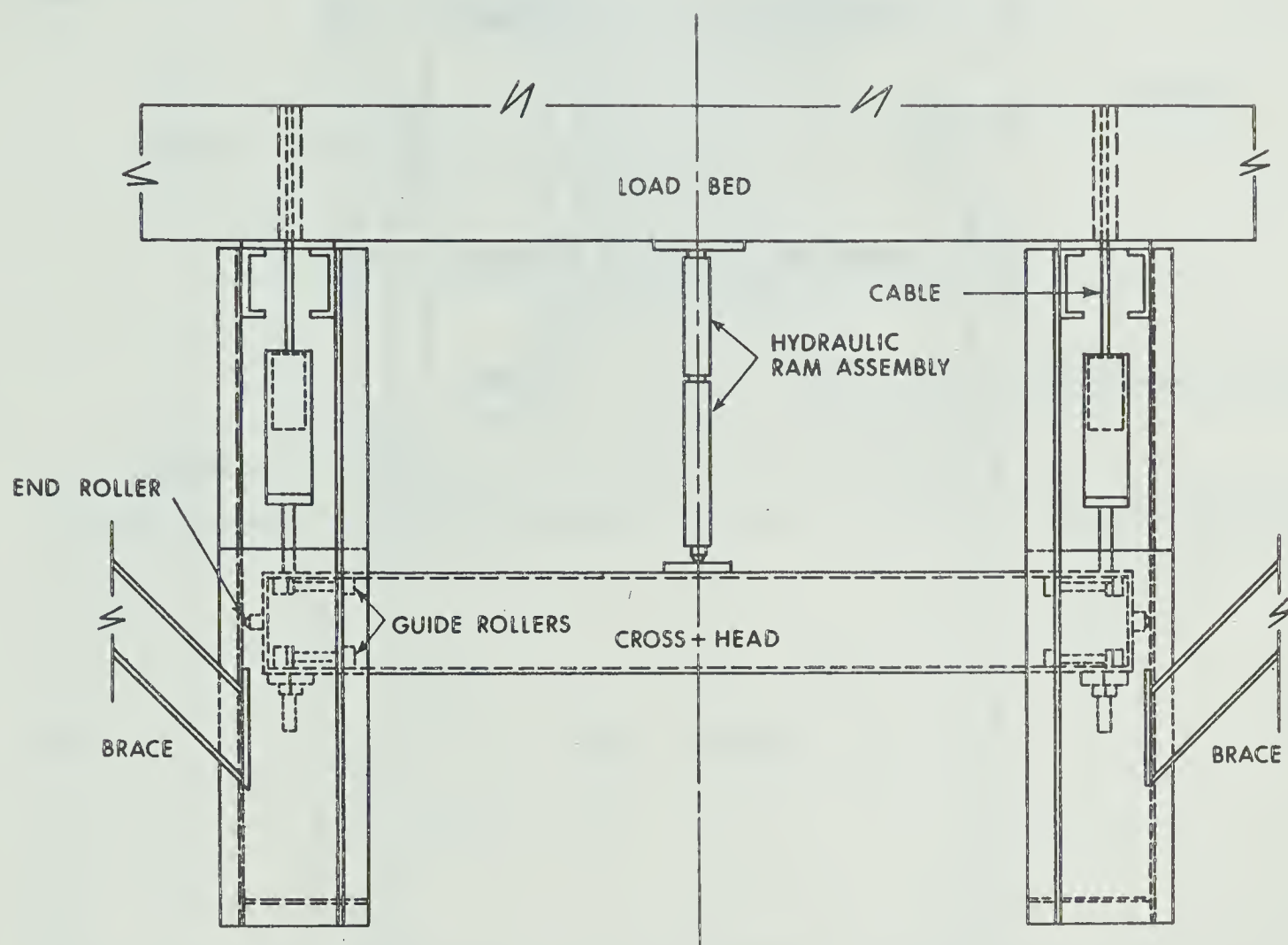


FIG. 3.8 TORSIONAL LOADING EQUIPMENT

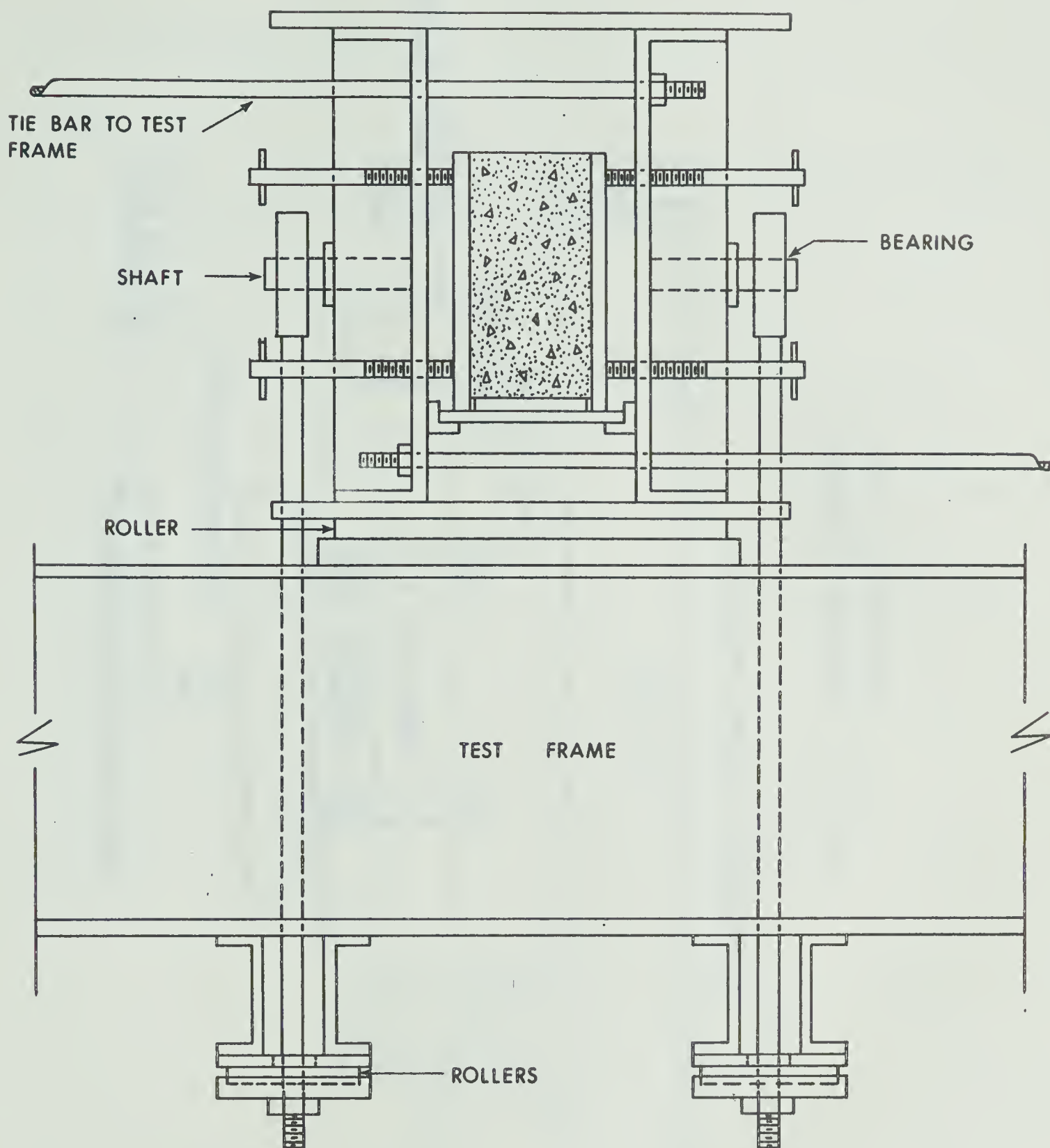


FIG. 3.9 FIXED HEAD

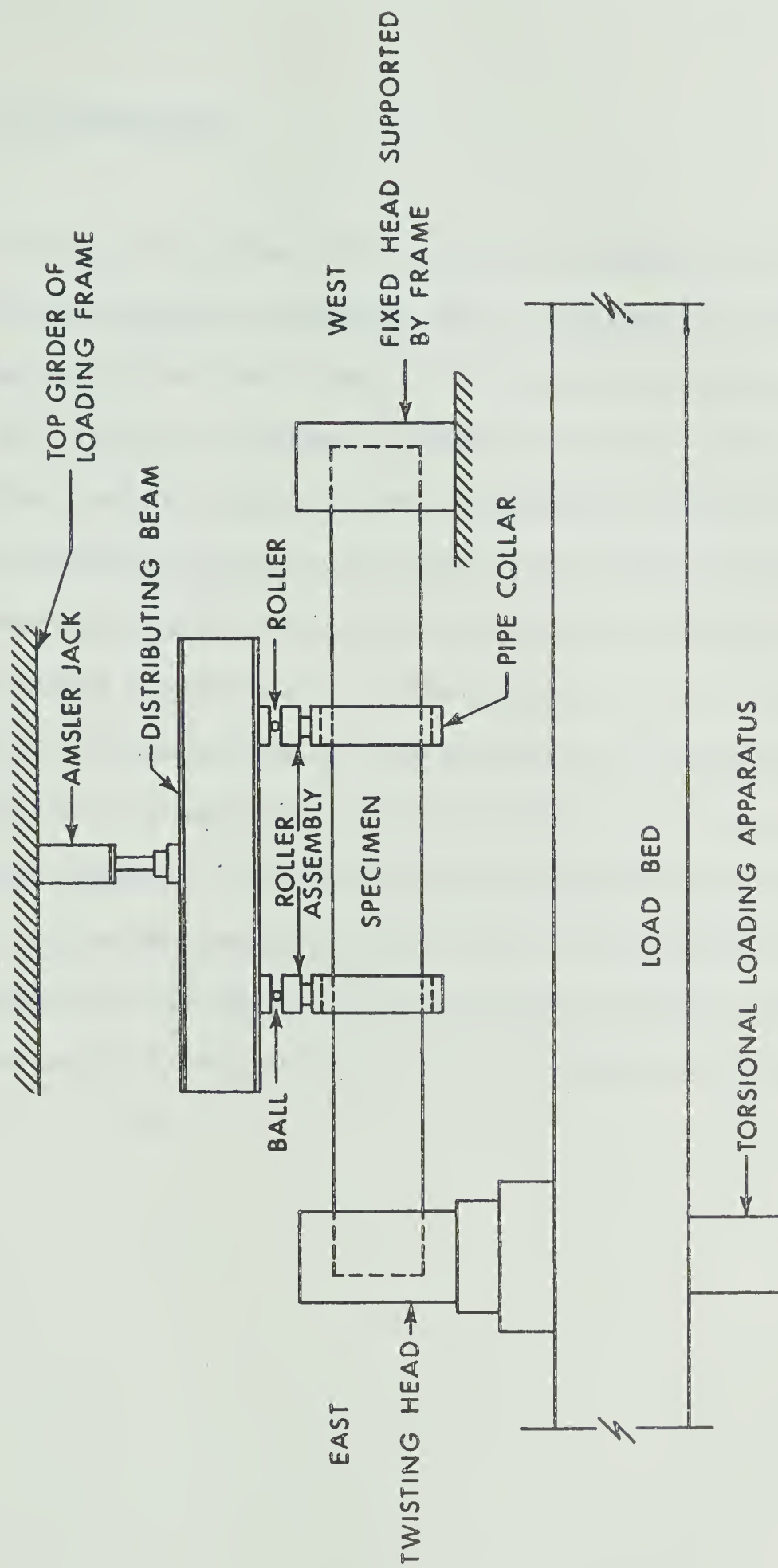


FIG. 3.10 EQUIPMENT ARRANGEMENT
FOR COMBINED LOADING

3.4 TEST PROCEDURE

The day of the test final mechanical gauge point readings were taken on the test beam thus obtaining the time dependent strains which occurred during the period from release of the strand to testing. The specimen was then placed in the test equipment and tested to failure.

The load was applied to the specimen in a series of increments. The relationship between the bending load increment and the torque load increment was controlled by the ϕ ratio for the beam and these increments were applied simultaneously to the specimen. When the specimen reached a critical stage such as initial cracking or ultimate load, the magnitude of the load increment was reduced to enable more data to be collected in these regions. The specimens were deformed beyond the load stage at which peak values were observed. After the application of each load increment, and the reading of the deflection gauges, the twistmeters, and the reinforcement strains, the crack pattern formation was marked.

CHAPTER IV

PRESENTATION OF TEST RESULTS

4.1 INTRODUCTION

The following sections indicate the manner in which the test results were obtained. The detailed results including tabulated readings after each load increment, moment-deflection curves, torque-twist curves and interaction diagrams are presented in Appendix A.

4.2 MOMENT-DEFLECTION RELATIONSHIPS

The location of the deflection gauges is illustrated in Figure 3.1. These gauges consisted of scales graduated in 0.01 in. increments and suspended at mid-depth of each specimen by a bracket clamped to the test specimen. These scales, 3 on each side of the specimen, were read after each load increment by using two precise levels located one on each side of the specimen. The readings from each set of gauges were averaged giving the deflection of the center of the specimen.

The bending moment was calculated as the product of one half the total transverse load and the distance from one load point to a support.

Once the bending moment and the deflections were known, the MOMENT-DEFLECTION curves were plotted. The curves as shown include only those points necessary to completely indicate the specimen's behaviour. This was done in order to obtain legible curves.

4.3 TORQUE-TWIST RELATIONSHIPS

The torsional moment was obtained by calculating, from load cell readings, the force in the twisting head cables and multiplying this force by the torsional moment arm of 72 in.

Two rotation gauges, located as shown in Figure 3.1, gave the corresponding twist for any torque load. The readings of the west gauge were subtracted from those of the east gauge yielding the angle change between them. This angle change was then divided by the distance between the rotation gauges to obtain the angle of twist in radians per inch.

The results are plotted in the form of TORQUE-TWIST curves. In order to obtain legible curves only those points necessary to define the behaviour of the specimen have been plotted.

4.4 EFFECTIVE PRESTRESS FORCE

The values of the effective prestress force indicated in Table 3.3 were obtained by subtracting, from the strand force values prior to release, the change in strand force due to elastic shortening and time dependent strains. The method of evaluating the elastic shortening and time dependent strains was discussed in Articles 3.2 and 3.4 respectively.

4.5 INTERACTION DIAGRAMS

An interaction diagram for torsion and flexure indicates the influence that one effect has on the other. Interaction diagrams have been

plotted in dimensional and non-dimensional form for both cracking and ultimate loads. These diagrams are shown in Appendix A.

The values of experimental cracking torque were obtained from the TORQUE-TWIST curves. The cracking torque is that value of torsional moment at which the TORQUE-TWIST curve changes abruptly from its initial straight line portion to a flatter curve. The value of bending moment which corresponded to a cracking torque and its associated ϕ value was considered to be the flexural cracking moment.

The ultimate torsional and flexural moments are the related maximum values as obtained during the test of any one beam.

For completeness the values used in plotting the interaction diagrams are also tabulated in Table A.13.

4.6 REINFORCEMENT STRAINS

Reinforcement strains were measured on only the transverse and longitudinal non-prestressed steel. No attempt was made to measure the strains in the prestressing strands because no technique could be devised which would yield satisfactory results. The reinforcement strains are tabulated in the tables in Appendix A, and a STRESS-STRAIN curve for the reinforcement is given in Figure 3.3.

Except for an indication of the stress level in the reinforcing steel throughout the stages of loading, no other use was made of the steel strain information.

CHAPTER V

DISCUSSION

5.1 INTRODUCTION

The behaviour of the test beams and the comparison of test results is described in this chapter. Two stages of behaviour were observed. These have been designated as pre-cracking and post-cracking behaviour. The termination of pre-cracking behaviour is indicated by an abrupt change in slope of the MOMENT-DEFLECTION curve and/or the TORQUE-TWIST curve. These curves, along with INTERACTION DIAGRAMS, are given in Appendix A.

5.2 BEHAVIOUR OF CONCENTRICALLY PRESTRESSED BEAMS

5.2.1 GENERAL BEHAVIOUR

These beams exhibited both stages of behaviour. In all beams except BEAM 102 the pre-cracking behaviour was terminated by the formation of a major crack or cracks which reduced the size and shape of the effective section thus causing an immediate reduction in stiffness. BEAM 102 exhibited a gradual transition from initially elastic pre-cracking behaviour to post-cracking behaviour. All beams exhibited considerable ductility due to the presence of mild steel reinforcement.

5.2.2 INDIVIDUAL BEAM BEHAVIOUR

BEAM 101 ($\phi = 0$)

This beam was tested in pure flexure. Even after some cracking had occurred, the behaviour remained essentially elastic up to about 60% of ultimate load. This behaviour is indicated by the very flat portion of the beam's MOMENT-DEFLECTION curve at approximately 270 IN.KIP. Further deformation resulted in an increase in load capacity to an ultimate load. The large deflections associated with this ultimate load indicate the beam possessed considerable ductility. This is also indicated by the general shape of the MOMENT-DEFLECTION curve which reflects the material properties of the non-prestressed reinforcing steel. Finally, failure of the beam occurred when the beam's compression zone failed after extreme yielding of the tensile reinforcement. The crack pattern is shown in Figure B.1.

BEAM 102 ($\phi = 1/3$)

This beam was subjected to a combined load. Both the MOMENT-DEFLECTION and the TORQUE-TWIST curves indicated a gradual change from initial elastic behaviour in the pre-cracking stage to ductile behaviour in the post-cracking stage. The termination of the pre-cracking stage was not clearly shown on either curve.

The ultimate failure was not characteristic of either a flexural failure or a torsional failure. It appeared to be transition type of

failure. Further deformation did not decrease either the torsional or the flexural capacity.

The crack pattern began as flexural cracks inclined at 90° to the longitudinal axis of the beam. With increasing load this inclination decreased until the cracks terminated in torsional cracks inclined at approximately 40° to the longitudinal axis of the beam. The crack pattern was similar to that shown for Beam 122 in Figure B.3.

BEAM 103 ($\phi = 3/4$)

The behaviour of this beam, also under combined load, clearly indicated both pre-cracking and post-cracking stages of behaviour. The termination of pre-cracking behaviour is evident from the shape of the TORQUE-TWIST curve which indicates that major cracking occurred at approximately 70% of the ultimate torque load. Beyond this point, during the post-cracking stage, the capacity of the beam increased up to ultimate load. When subjected to further deformations the flexural capacity remained constant while the torsional capacity decreased as indicated in Figures A.1 and A.5. This decrease in torsional capacity was associated with yielding of the transverse reinforcement at the ultimate torque load.

The failure of the beam was definitely controlled by torsion. The failure surface was inclined at approximately 43° to the longitudinal axis of the beam.

BEAM 104 ($\phi = 4/3$)

The initial behaviour of this beam, under combined load, was similar to BEAM 103 except that the pre-cracking stage was terminated at about 82% of ultimate load. Also, in contrast to BEAM 103, deformations beyond the ultimate load values appeared to produce no decrease in load capacity. This is illustrated by the shape of the MOMENT-DEFLECTION and TORQUE-TWIST curves as shown in Figures A.2 and A.5.

The failure was controlled by torsion with the failure surface inclined about 40° to the longitudinal axis of the beam. The crack pattern is shown in Figure B.1.

BEAM 105 ($\phi = 3$)

The combined load behaviour of this beam was essentially elastic to about 80% of its ultimate capacity. This pre-cracking stage of behaviour was terminated by major torsional cracking and is indicated on the TORQUE-TWIST curve by an abrupt decrease in slope at approximately 93 IN.KIP. The behaviour during the post-cracking region showed an increase in capacity to ultimate load at which time the transverse reinforcement yielded. Further deformations caused the torsional capacity to reduce while the flexural capacity remained constant.

The failure was torsional in nature and the inclination of the failure surface was about 39° to the longitudinal axis of the beam. The crack pattern is shown in Figure B.2.

BEAM 106 ($\phi = \infty$)

This beam was tested in pure torsion. The pre-cracking behaviour was elastic up to 71% of the beam's ultimate load. The post-cracking behaviour was the same as for preceding beams with the exception that the torsional capacity decreased more rapidly, after ultimate load, than any preceding beam. The associated rotations were very large.

The failure was characterized by the further opening of cracks which had formed at lower loads. The failure surface was inclined at about 38° to the longitudinal axis of the beam. The crack pattern is shown in Figure B.2.

5.3 BEHAVIOUR OF ECCENTRICALLY PRESTRESSED BEAMS

5.3.1 GENERAL BEHAVIOUR

These beams exhibited both pre-cracking and post-cracking stages of behaviour. With two exceptions, their general behaviour pattern was similar to that of the concentrically prestressed beams. The two exceptions were the magnitude and variation of the load terminating the pre-cracking stage and the magnitude and variation of the ultimate load attained during the post-cracking stage. These beams also possessed considerable ductility due to the presence of mild steel reinforcement.

5.3.2 INDIVIDUAL BEAM BEHAVIOUR

BEAM 121 ($\phi = 0$)

This beam, tested in pure flexure, showed no distinct change from initially elastic pre-cracking behaviour to ductile post-cracking behaviour. Generally the shape of the beam's MOMENT-DEFLECTION curve was smooth. However, there was a small break in the curve at about 500 IN.KIP. This was assumed to be the load terminating the pre-cracking stage. Additional deformation beyond this point produced a much lower rate of increase in load capacity indicating ductile behaviour.

Prior to failure both the top and bottom reinforcement had yielded. The crack pattern is shown in Figure B.3. Finally, the beam failed in an under-reinforced manner at about 565 IN.KIP.

BEAM 122 ($\phi = 1/3$)

This beam was subjected to a combined load. The initially elastic pre-cracking stage of behaviour was terminated at about 66% of the ultimate torque capacity of the beam. During the initial post-cracking stage, additional deformations produced an increase in capacity to ultimate load. Further twist deformations resulted in a decrease in torsional capacity while the flexural capacity remained constant at its ultimate value. The transverse steel was observed to yield in the vicinity of ultimate load. These behavioural patterns are indicated by the MOMENT-DEFLECTION and TORQUE-TWIST curves for the beam.

The failure surface occurred at previously defined cracks. These cracks were initially vertical but they had a final inclination of about 40° to the longitudinal axis of the beam. The crack pattern is shown in Figure B.3.

BEAM 123 ($\phi = 3/4$)

This beam, under combined load, gave a reasonable indication of the termination of elastic behaviour. This occurred at approximately 77% of its ultimate torque load. Beyond this point both the TORQUE-TWIST curve and the MOMENT-DEFLECTION curve show a reduced slope. The ultimate load condition was controlled by torsion. Beyond ultimate load, additional deformations reduced the torsional capacity while the flexural capacity remained constant at its ultimate value.

At failure, the transverse reinforcement had yielded. The failure surface occurred at crack locations formed prior to ultimate load and had an inclination of about 38° to the longitudinal axis of the beam as shown in Figure B.4.

BEAM 124 ($\phi = 4/3$)

This beam, subjected to combined load, exhibited a definite change from the initially elastic pre-cracking stage to the ductile post-cracking stage. This occurred at the major torsional cracking load of about 117 IN.KIP. This cracking torque was the highest cracking torque observed in both series and will be discussed further in Section 5.5.

After cracking, the stiffness was reduced and the applied torsional load was resisted by the action of the transverse steel and whatever concrete section was still effective. This is illustrated by the relatively flat portion of the TORQUE-TWIST curve in the vicinity of 120 IN.KIP. and by the increase in transverse reinforcement strains after major cracking. Further deformation caused the beam's capacity to increase to an ultimate load which was the highest of any beam tested.

In the vicinity of ultimate load the beam showed a tendency toward negative deflections. This tendency, indicated by the MOMENT-DEFLECTION curve, will be dealt with in Section 5.4.

Beyond ultimate load, the flexural moment was maintained at the ultimate value while further twist deformations were applied. This resulted in a decrease in torsional capacity and loading was then terminated.

The beam failed in torsion with the failure surface being located at previously formed cracks inclined at about 35° to the longitudinal axis of the beam. The crack pattern is shown in Figure B.4.

BEAM 125 ($\phi = 3$)

The behaviour of this beam was identical to that of BEAM 124 except for three aspects. The first of these is a decrease in torsional stiffness, immediately after cracking, which was not as great as that for BEAM 124. The second concerns the negative deflection of the total beam in the region of post-cracking behaviour and the third is indicated by the fact that failure occurred by further opening of previously formed cracks which were inclined at approximately 41° to the longitudinal axis

of the beam as shown in Figure B.5.

The relative decrease in stiffness after reaching the cracking torque is not significant enough to warrant any major discussion as the value of the ultimate load was almost the same for the last four beams of this type. The negative deflection problem will be dealt with in Section 5.4 and the increase in crack angle inclination will be considered in Section 5.5.

BEAM 126 ($\phi = \infty$)

This beam was tested in pure torsion. The behaviour was elastic up to approximately 73% of its ultimate torsional capacity. The post-cracking behaviour was characteristic of all beams in this series. The torsional stiffness was greatly reduced by major cracking but the beam did sustain an increase in load from the cracking load up to ultimate load. The associated deformations were large and any attempt to further deform the beam after ultimate load resulted in a decrease in the torsional resistance of the beam. The transverse reinforcement was observed to be well into the yielded range at ultimate load.

The failure occurred by expansion of a previously defined crack pattern which was oriented at 45° to the longitudinal direction of the beam as shown in Figure B.5.

Vertical deflections of this beam were not measured however, had they been observed, they assuredly would have been negative.

5.4 GENERAL DISCUSSION

A certain degree of elastic (pre-cracking) behaviour and post-cracking behaviour was observed in each beam tested. If the early stages of elastic behaviour are compared it is found that all beams exhibited elastic behaviour which was terminated by a cracking situation either flexural or torsional. Once this cracking had progressed to the point where a great deal of the applied load was being resisted by the reinforcement the beams exhibited a reduction in stiffness characterized by a reduction in the slopes of both the MOMENT-DEFLECTION and the TORQUE-TWIST curves. The capacity of all beams increased beyond the termination of elastic behaviour to ultimate values of load ranging between 40 to 60% more than the cracking load. At ultimate load, the reinforcement corresponding to the type of failure yielded. If the failure was controlled by flexure, the bottom reinforcement yielded. If the failure was controlled by torsion, the transverse steel yielded. The transition range of behaviour, where neither flexure nor torsion appeared to control the failure, was characterized by a constant reduction in slope of both the MOMENT-DEFLECTION curve and the TORQUE-TWIST curve from initial loading up to ultimate load. This type of behaviour was observed for BEAMS 102 and 122. All beams except BEAMS 102 and 104 appeared to suffer a decrease in torsional capacity at rotational deformations larger than those at ultimate load. Had further deformations been applied to BEAMS 102 and 104, they too would have exhibited a decrease in torsional capacity. The flexural capacity of all beams was observed to stabilize at the ultimate value obtained during the test. This occurred regardless

of any attempt to apply further torsional deformations.

Of interest is the behaviour of the vertical deflections and the top longitudinal steel during the post-cracking stage for all beams with a values of ϕ greater than 1. In all these beams the top longitudinal reinforcement (strain gauge #6) exhibited tensile strains at or shortly after the termination of the pre-cracking stage of behaviour. This would indicate a trend towards upward deflection. However, BEAM 104 still indicated a downward deflection at ultimate load. BEAMS 105, 124, and 125 all indicated a tendency to deflect upward against the transverse load. This indicates a change in the mode of failure as defined by McMullen (5).

5.5 EFFECT OF PRESTRESSING

5.5.1 CONCENTRIC PRESTRESSING

The effect of prestressing a beam concentrically is to increase the effective tensile strength of the concrete by an amount equal to the effective prestress force divided by the cross-sectional area. No specimens are available for a direct comparison of the relative strength of non-prestressed and prestressed beams but a knowledge of elementary prestressing theory would indicate the stated effect to be true. The upper limit of effective concentric prestressing is that level of prestress which would cause the member to suffer a reduced combined load capacity due to a compression failure.

5.5.2 EFFECT OF ECCENTRIC PRESTRESSING AT AN ECCENTRICITY RATIO OF 0.167

The test results indicate that the eccentrically prestressed beams possessed a greater capacity for combined load than the equivalent concentrically prestressed beams. As explained by Mukherjee (6), increased prestress is beneficial in increasing the torsional capacity up to a level of prestress which changes the failure mode from a principal tensile failure to a compression failure. Eccentric prestressing, combined with a flexural moment of the opposite sense, has the general effect of creating a higher level of prestress. The test results of the eccentrically prestressed beams indicate the most effective amount of eccentric prestressing to be a level of prestress such that the internal moment due to the prestress force balances the external moment due to transverse loading. This creates an effective uniform prestress and increases the combined load capacity of the beam. It appears that any other combination of internal prestressing moment and external flexural moment gives this type of beam a reduced combined load capacity. This is evidenced by BEAM 124 possessing the highest values of cracking and ultimate torque of any beam tested for this section of the study. The value of the flexural moment corresponding to the cracking torque was approximately 87 in.kip. This flexural moment slightly over-balanced the internal prestressing moment of about 73 in.kip. but it was close enough to the prestressing moment to illustrate the validity of the preceding discussion.

5.5.3 GENERAL EFFECT OF PRESTRESSING

The general effect of prestressing is that of increasing the capacity of a reinforced concrete beam subjected to combined loading. One adverse effect however, is that a reduction in the amount of post-cracking behaviour is possible with increasing levels of prestress. Thus, to ensure ductile behaviour, at loads beyond the cracking load, the level of prestressing must be limited.

5.6 EFFECT OF THE ϕ RATIO

The best method of illustrating the effect of the ϕ ratio is by means of Interaction Diagrams. Interaction Diagrams have been constructed for both cracking and ultimate load stages and for both concentric and eccentrically prestressed beams. These diagrams form a complete summary of the results of the twelve beams tested and are presented in Appendix A.

5.6.1 CONCENTRICALLY PRESTRESSED BEAMS

The behaviour of the beams which were concentrically prestressed indicated that at no time was the torsional capacity increased by the presence of bending moments less than or equal to the flexural cracking moment nor was the flexural capacity increased by torsional moments less than or equal to the torsional cracking moment. This is well illustrated by the shape of the Interaction Curves in Appendix A which

indicate that at any value of ϕ the respective capacities are reduced.

5.6.2 ECCENTRICALLY PRESTRESSED BEAMS

The general behaviour of these beams indicates that the presence of flexural moment did increase the torsional capacity but the presence of torsional moment reduced the flexural capacity. The Interaction Curves in Appendix A indicate the optimum ϕ value, with respect to torsional capacity, to be approximately $\phi = 4/3$. No value of ϕ was observed which yielded flexural moments higher than that obtained for BEAM 121 tested in pure flexure.

CHAPTER VI

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

This chapter presents a summary of the information in the preceding chapters, general conclusions resulting from the study, and recommendations for further studies of this nature.

6.2 SUMMARY

Twelve reinforced rectangular concrete beams were tested for this thesis. These beams were fabricated from commonly used materials and were divided into two series. The first series, BEAMS 101 to 106 inclusive, was concentrically prestressed. The second series, BEAMS 121 to 126 inclusive, was eccentrically prestressed at an eccentricity ratio of 0.167.

All beams exhibited two stages of behaviour. These are designated as the pre-cracking stage and the post-cracking stage. The pre-cracking stage was initially elastic and was terminated by major cracking. This was usually indicated by an abrupt change in slope of either the MOMENT-DEFLECTION curves, the TORQUE-TWIST curve, or both curves for the beam. The post-cracking stage exhibited considerable ductility due to the presence of the mild steel reinforcement. During this stage all beams exhibited an increase in capacity to ultimate load. Additional deformations beyond ultimate load caused a majority of the beams to exhibit

a decreased torsional capacity and a constant flexural capacity.

The test results have been presented in the form of discussion, tabulated values, MOMENT-DEFLECTION curves, TORQUE-TWIST curves, and INTERACTION DIAGRAMS in Appendix A. Special consideration should be given to the Interaction Diagrams. They have been constructed in both dimensional and non-dimensional form for both cracking and ultimate load conditions. These diagrams give a good summary of the general test results indicating the effects of both types of prestressing and the torsional moment to flexural moment (ϕ) ratio.

6.3 CONCLUSIONS

The following conclusions are based on the test results of a limited sample of twelve beams. This fact must be remembered when interpreting the conclusions.

From the test results, it is concluded that:

(1) Concentric prestressing increases the effective tensile strength of the concrete thus increasing the beam's cracking capacity for combined load.

(2) Eccentric prestressing, at an eccentricity ratio of 0.167, increases the beam's cracking capacity for combined load. The greatest increase in cracking capacity appears to occur when the internal moment due to prestressing is equal and opposite to the external flexural moment thus creating a uniform prestress condition.

(3) Generally, the effect of prestressing is beneficial provided the level of prestress is not great enough to change the failure

mode from a principal tensile mode to a compressive failure mode.

The results of varying the torsional moment to flexural moment (ϕ) ratio indicate that:

(1) For concentrically prestressed beams with equal amounts of top and bottom reinforcement, the presence of any flexural moment reduces the torsional capacity and the presence of any torsional moment reduces the flexural capacity. This appears to be true for both cracking and ultimate stages of behaviour.

(2) For eccentrically prestressed beams with an eccentricity ratio of 0.167 and equal amounts of top and bottom reinforcement:

- (a) any value of ϕ greater than 0.63 yielded an increase in both cracking torque and ultimate torque capacities.
- (b) any value of ϕ less than 0.63 yielded a decrease in both cracking torque and ultimate torque capacities.
- (c) any value of ϕ greater than ZERO yielded a decreased flexural capacity.

6.4 RECOMMENDATIONS

The following recommendations are made for the benefit of future investigations.

(1) Instrumentation of all longitudinal reinforcement should be attempted in order to clarify the mode of failure.

(2) Future prestressed specimens should be fabricated so that they are identical, except for the prestressing, to previously tested non-prestressed specimens for which the combined load test data

is available. This would enable definite comparable conclusions to be made as to the effect of prestress on combined load capacity.

(3) More test data is required for ϕ values between $1/3$ and 0 since a considerable amount of present construction contains members subjected to high flexural moments and relatively low torsional moments. The suggested ϕ values for these tests are ϕ equal to 0.20 and ϕ equal to 0.10.

(4) Vertical deflections must be observed on every test specimen as the mode of failure may be indeterminate for a high torsion - low flexure condition. A tendency for vertical deflections, opposite in direction to the applied transverse load, indicates a possible change in the failure mode.

(5) The scope of experimental tests should be extended to include the effects of shear.

(6) An attempt should be made to develop a means of observing the behaviour of the prestressing forces during a combined load test. As the prestress forces are moved outward towards the periphery of the cross-section, (i.e. increasing eccentricity ratios) the need for these observations increases.

LIST OF REFERENCES

1. Nylander, H., "Vridning och vridningsin-spänning vid betongkonstruktioner," Statens Kommitté för Byggnadsforskning, Stockholm, Bulletin No. 3, 1945.
2. Cowan, H.J. and Armstrong, S., "Experiments on the Strength of Reinforced and Prestressed Concrete Beams and of Concrete-Encased Steel Joists in Combined Bending and Torsion," Magazine of Concrete Research, Vol. 7, No. 19, March 1955.
3. Zia, P., "Torsional Strength of Prestressed Concrete Members," Journal of the American Concrete Institute, Vol. 32, No. 10, April 1961.
4. Swamy, N., "Prestressed Concrete Hollow Beams Under Combined Bending and Torsion," Prestressed Concrete Journal.
5. McMullen, A.E., "Torsional Strength of Rectangular Reinforced Concrete Beams Subjected to Combined Loading," Ph.D. Thesis, University of Alberta, Civil Engineering, 1967.
6. Mukherjee, P.R., "Ultimate Torsional Strength of Plain, Prestressed and Reinforced Concrete Members of Rectangular Cross Section," Ph.D. Thesis, West Virginia University, 1967.
7. Kemp, E.L., Sozen, M.A., Siess, C.P., "Torsion in Reinforced Concrete," Civil Engineering Studies, Structural Research Series No. 226, University of Illinois, Urbana, Illinois, 1961.

APPENDIX A

TEST RESULTS

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER. IN. x 10 ⁶	DEFLECTIONS IN. x 10 ³			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	-	12.1	-	0	0	0	0	0	0	0	0	0
1	-	21.6	-	0	0	0	-	3	0	+13	-	11
2	-	39.2	-	0	5	0	-	2	0	32	-	37
3	-	59.5	-	0	10	10	-	1	1	63	-	63
4	-	86.5	-	10	20	15	-	3	-	105	-	104
5	-	113.4	-	25	35	25	-	3	-	175	-	159
6	-	147.0	-	45	65	45	-	-13	-17	430	-	242
7	-	174.0	-	65	85	70	-	8	-19	648	-	323
8	-	187.7	-	85	105	85	-	7	-13	770	-	369
9	-	201.0	-	100	125	105	-	7	-6	930	-	431
10	-	228.0	-	140	175	135	+25	+24	+24	1265	-	549
11	-	268.5	-	210	270	215	68	50	50	1559	-	565
12	-	273.5	-	265	335	260	54	40	40	1828	-	667
13	-	278.5	-	320	400	320	72	62	62	1970	-	742
14	-	282.0	-	380	480	375	111	75	75	1993	-	835
15	-	302.5	-	410	520	405	84	48	48	2542	-	705
16	-	322.5	-	440	560	435	100	54	54	2689	-	758
17	-	343.0	-	480	610	475	119	61	61	2815	-	829
18	-	356.3	-	530	670	525	148	76	76	2858	-	898
19	-	376.5	-	645	815	635	221	95	95	2916	-	1028
20	-	390.0	-	735	920	720	266	124	124	5385	-	1157
21	-	404.0	-	1015	1295	1040	322	166	166	-	-	-3765
22	-	417.0	-	1155	1470	1175	352	202	202	-	-	-
23	-	424.0	-	1255	1585	1275	358	219	219	-	-	-
24	-	437.3	-	1450	1805	1500	397	271	271	-	-	-

TABLE A.1 BEAM 101

NOTE: DEFLECTIONS POSITIVE DOWN
POSITIVE STRAIN INDICATES TENSION

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10 ⁶	DEFLECTIONS IN. x 10 ³			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	12.2	0	0	0	0	0	0	0	0		0
1	13.5	40.5	15	15	10	5	0	0	6	-		- 52
2	18.0	54.0	16	20	15	15	-1	-1	13	-		- 60
3	22.5	67.5	22	20	25	20	-1	-1	14	-		- 66
4	27.0	81.0	20	30	30	30	-3	-3	18	-		- 87
5	31.5	94.5	36	40	40	35	-4	-4	19	-		-116
6	36.0	108.0	37	50	50	40	-4	-4	22	-		-142
7	40.5	121.5	48	55	65	55	-6	-6	22	-		-171
8	45.0	135.0	54	70	80	70	-1	-1	24	-		-197
9	49.5	148.5	65	80	90	85	+6	+6	27	-		-235
10	54.0	162.0	81	95	115	95	9	9	52	-		-270
11	58.5	175.5	104	120	140	115	22	22	72	-		-322
12	63.0	189.0	141	150	170	150	52	52	97	-		-379
13	67.5	202.5	160	175	205	175	63	63	107	-		-435
14	72.0	216.0	217	210	255	210	128	128	120	-		-483
15	76.5	229.5	242	235	290	235	156	156	120	-		-523
16	81.0	243.0	291	270	330	270	210	210	114	-		-565
17	85.5	256.5	343	310	380	310	280	280	113	-		-620
18	90.0	270.0	555	400	495	395	690	690	63	-		-687
19	92.3	276.8	603	430	530	425	713	713	23	-		-713
20	94.5	283.5	667	465	585	465	740	740	+ 35	-		-750
21	96.8	290.3	888	565	705	560	861	861	105	-		-825
22	99.0	297.0	980	620	775	615	910	910	137	-		-877
23	101.3	303.8	1169	700	880	700	964	964	196	-		-941
24	103.5	310.5	1280	755	955	755	1017	1017	210	-		-998
25	105.8	317.3	2033	1250	1510	1095	1211	1211	340	-		-967

TABLE A.2 BEAM 102

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIAN PER IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	12.2	0	0	0	0		0	0	0	0	0
1	20.2	27.0	19	5	10	5		-	0	2	22	- 23
2	30.4	40.5	31	10	15	10		-	0	3	48	- 44
3	40.5	54.0	43	15	20	20		-	2	4	76	- 83
4	45.6	60.8	48	15	20	25		+	4	-	90	- 90
5	50.6	67.5	54	20	30	25		3	6	1	107	- 98
6	55.6	74.3	65	25	35	30		3	8	0	127	- 112
7	60.7	81.0	68	25	40	30		8	10	3	147	- 128
8	70.9	94.5	90	40	50	45		14	14	7	198	- 157
9	81.0	108.0	124	50	65	60		44	25	0	247	- 192
10	86.0	114.8	147	55	80	65		105	110	0	277	- 202
11	91.1	121.5	250	80	110	90		473	327	- 17	584	- 210
12	96.1	128.3	346	105	140	115		618	640	+116	812	- 204
13	101.2	135.0	412	120	160	135		784	825	478	1032	- 226
14	106.3	141.8	538	145	190	155		955	929	627	1324	- 226
15	111.4	148.5	677	165	215	185		1143	1075	763	1727	- 232
16	116.4	155.3	873	185	245	215		1217	1185	1025	2092	- 190
17	121.5	162.0	1066	220	285	245		1415	1276	1247	2394	- 153
18	125.1	168.8	-	-	-	-		1713	1402	1377	2487	- 518
19	108.6	-	1750	315	415	325		-	-	-	-	-
20	77.2	209.3	-	-	-	-		-	-	-	-	-

TABLE A.3 BEAM 103

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIAN PER IN. $\times 10^6$	DEFLECTIONS $\text{IN.} \times 10^3$			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	12.2	0	0	0	0	0	0	0			0
1	16.2	12.2	8	5	0	5	- 2	- 2	0			3
2	27.0	20.3	20	10	5	5	+ 1	+ 1	2			9
3	32.4	24.3	23	10	5	5	0	0	2			- 18
4	37.8	28.4	30	5	10	10	5	3	5			- 22
5	43.2	32.4	34	10	10	10	4	6	8			- 28
6	48.6	36.5	42	10	10	10	8	8	8			- 36
7	54.0	40.5	50	15	15	15	8	12	14			- 44
8	59.4	44.6	59	20	15	15	15	20	20			- 50
9	64.8	48.6	70	20	20	20	25	25	28			- 58
10	70.2	52.7	74	15	20	25	44	33	34			- 65
11	75.6	56.7	87	20	25	25	57	44	38			- 75
12	81.0	60.8	95	25	30	30	75	51	41			- 81
13	86.4	64.8	110	25	35	30	91	62	49			- 92
14	91.8	68.9	126	30	35	35	115	83	58			- 97
15	97.2	72.9	149	35	45	40	157	79	116			- 105
16	102.6	77.0	200	45	50	45	237	100	370			- 114
17	108.0	81.0	383	60	80	70	408	736	856			- 31
18	111.6	83.7	479	75	90	80	518	986	958			+ 33
19	115.2	86.4	566	80	105	90	558	1200	1051			71
20	118.8	89.1	657	90	115	100	660	1478	1154			86
21	122.4	91.8	798	110	135	115	778	1752	1288			163
22	126.0	94.5	913	115	145	115	887	1960	1390			192
23	129.6	97.2	1070	130	160	130	979	2242	1486			318
24	133.2	99.9	1683	160	165	165	1031	6629	1474			688

TABLE A.4 BEAM 104

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10 ⁶	DEFLECTIONS IN. x 10 ³			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	12.2	0	0	0	0	0	0	0	0	0	0
1	36.5	12.2	39	- 5	- 5	- 5	- 3	- 4	9	-11	-11	- 11
2	44.6	14.9	48	- 5	- 5	0	- 7	-12	-12	- 4	- 4	- 14
3	52.7	17.6	59	- 5	0	0	- 7	-12	-12	0	0	- 26
4	60.8	20.3	79	- 5	0	5	- 1	+ 4	- 7	7	7	- 27
5	68.9	23.0	88	0	5	5	- 1	2	- 4	8	8	- 49
6	77.0	25.7	102	0	0	5	+ 9	14	+ 4	22	22	- 44
7	85.1	28.4	120	0	5	10	23	40	19	36	36	- 51
8	93.2	31.1	157	0	5	10	78	127	36	59	59	- 38
9	101.3	33.8	282	0	10	15	116	208	29	75	75	+176
10	109.4	36.5	575	10	25	30	1029	1340	557	497	497	275
11	113.4	37.8	660	20	35	30	1160	1488	670	607	607	304
12	117.5	39.2	719	25	40	30	1315	1618	780	693	693	317
13	121.5	40.5	885	30	50	45	1498	1896	952	894	894	365
14	125.6	41.9	1049	40	55	45	1708	5605	1050	1117	1117	420
15	129.6	43.2	1288	50	70	55	2016	10290	1145	1828	1828	461
16	133.7	44.6	1710	60	75	70	2100	9958	1285	2466	2466	475
17	137.7	45.9	-	-	-	-	-	-	-	-	-	-
18	105.9	45.9	3580	155	70	125	1930	7558	1324	2780	2780	127

TABLE A.5 BEAM 105

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	TWIST RADIANS PER IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$		REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER WEST	1	2	3	4	5	6
0	0	-	0			0		0	0		0
1	11.2	-	11			2		1	0		2
2	22.5	-	25			3		2	2		4
3	30.2	-	31			4		3	2		4
4	38.1	-	37			5		3	3		2
5	45.8	-	43			7		2	4		4
6	53.7	-	56			11		1	13		5
7	61.5	-	68			21		1	24		1
8	69.4	-	78			31		2	34		0
9	77.3	-	95			50		1	48		1
10	85.2	-	107			88		2	58		7
11	93.0	-	132			146		15	70		20
12	100.8	-	161			221		18	85		34
13	108.6	-	378			410		517	66		345
14	112.5	-	443			580		630	204		422
15	116.5	-	570			793		743	724		525
16	119.6	-	676			935		813	878		584
17	122.8	-	752			1094		863	985		656
18	126.6	-	865			1210		961	1100		767
19	130.6	-	1012			1322		1107	1225		911
20	133.7	-	1206			1453		1266	1364		1176
21	135.3	-	1468			1604		1468	1527		1579
22	140.0	-	1733			1707		1655	1630		1850
23	140.8	-	2334			1673		1970	1736		2207
24	88.3	-	3314			1270		1922	1330		2120

TABLE A.6 BEAM 106

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10 ⁶	DEFLECTIONS IN. x 10 ³			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
				EAST	CENTER	WEST	1	2	3	4	5	6																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
0	-	12.2	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A.7 BEAM 121

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIAN PER IN. x 10 ⁶	DEFLECTIONS IN. x 10 ³			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	12.2	0	0	0	0	0	0	0	0	0	0
1	13.5	40.5	9	10	15	10	0	2	4	4	40	45
2	22.6	67.5	19	25	30	20	2	4	3	-	81	88
3	31.6	94.5	25	35	40	35	3	-	7	+	131	133
4	40.6	121.5	39	45	55	45	6	-	5	-	191	180
5	49.5	148.5	51	60	70	55	14	-	15	-	261	235
6	58.5	175.5	71	80	95	80	22	0	23	0	343	293
7	63.1	189.0	81	90	105	90	30	2	40	1	396	326
8	67.6	202.5	95	95	120	95	31	4	66	8	454	367
9	72.0	216.0	113	115	135	105	44	-	64	9	786	423
10	76.7	229.5	136	130	160	120	76	-23	96	8	933	468
11	81.0	243.0	172	145	175	145	175	-37	200	24	1128	511
12	85.5	256.5	211	170	205	165	251	-40	330	85	1316	547
13	90.2	270.0	273	195	235	185	366	-48	482	142	1471	585
14	94.5	283.5	340	225	270	225	506	-29	610	227	1614	620
15	99.0	297.0	408	245	315	245	591	-8	728	319	1726	632
16	103.5	310.5	468	280	345	280	656	163	809	400	360	667
17	108.0	324.0	496	305	380	305	724	311	881	473	332	697
18	112.8	337.5	608	340	415	335	817	427	989	568	405	723
19	117.0	351.0	755	385	475	380	914	620	1154	681	560	736
20	119.0	357.8	1035	445	550	430	1045	962	1300	782	622	656
21	121.8	364.5	1361	540	655	495	1056	1217	1409	831	656	875
22	117.2	371.3	1897	720	855	635	990	1528	1457	818	704	1248
23	106.0	378.0	2395	930	1055	775	969	1700	1378	723	500	1145

TABLE A.8 BEAM 122

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10 ⁶	DEFLECTIONS IN. x 10 ³			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	12.2	0	0	0	0	0	0	0	0		0
1	20.2	27.0	16	5	5	5	5	3	-	2		- 24
2	30.4	40.5	26	15	15	10	7	5	0	2		- 50
3	40.5	54.0	39	20	20	20	9	6	-	2		- 79
4	50.6	67.5	50	25	30	25	14	12	-	4		-112
5	60.8	81.0	67	30	35	30	23	20	+	5		-140
6	65.9	87.8	71	35	45	30	29	25	13	5		-156
7	71.0	94.5	81	40	45	35	34	30	18	8		-173
8	75.8	101.3	87	40	50	40	40	32	20	11		-195
9	81.1	108.0	101	45	55	45	54	45	32	16		-206
10	86.0	114.8	110	50	60	50	63	52	40	20		-225
11	91.1	121.5	123	55	65	50	71	61	57	28		-244
12	96.1	128.3	140	60	70	55	80	68	73	31		-265
13	101.2	135.0	161	70	75	65	100	88	248	42		-280
14	106.6	141.8	188	75	90	70	115	115	398	58		-303
15	111.2	148.5	234	85	100	85	160	125	542	243		-319
16	116.7	155.3	353	100	125	100	214	425	858	612		-323
17	121.8	162.0	479	120	145	115	422	740	1033	825		-285
18	123.8	164.7	567	135	160	130	823	922	1165	955		-245
19	125.8	167.4	632	140	170	140	964	1057	1226	1058		-210
20	127.3	170.1	687	145	175	145	1056	1134	1285	1111		-199
21	129.6	172.8	755	155	185	150	1118	1197	1337	1181		-177
22	131.9	175.5	807	160	190	160	1207	1273	1391	1243		-156
23	134.0	178.2	849	165	200	170	1254	1321	1428	1292		-140
24	138.0	183.6	947	180	215	180	1391	1442	1504	1377		-130
25	142.0	189.0	1018	190	210	190	1458	1523	1576	1467		-125
26	146.0	194.4	1146	205	230	205	1607	1680	1633	1596		-145
27	150.2	199.8	1355	225	245	220	1824	3263	1543	1801		- 70
28	147.6	205.2	1750	250	280	255	1972	4550	1553	2364		-133
29	144.0	207.9	2124	275	300	275	1916	4271	1601	2519		-263
30	137.5	213.3	-	-	-	-	-	-	-	-		-

TABLE A.9 BEAM 123

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIAN PER IN. x 10 ⁶	DEFLECTIONS IN. x 10 ³			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	12.2	0	0	0	0	0	0	0	0	0	0
1	16.2	12.2	-	5	0	- 5	0	0	6	4	0	1
2	36.0	27.0	25	+ 5	5	0	3	0	10	- 1	23	- 24
3	54.0	40.5	51	5	10	5	5	3	20	+ 9	44	- 48
4	72.0	54.0	76	10	20	10	15	21	40	18	69	- 75
5	81.0	60.8	96	20	25	15	26	27	55	27	82	- 86
6	90.0	67.5	110	20	25	20	32	47	72	34	99	- 94
7	99.0	74.3	132	25	35	25	74	65	100	48	118	- 99
8	108.0	81.0	166	25	35	30	169	83	138	60	141	- 100
9	117.0	87.8	217	35	55	35	501	266	294	460	265	- 44
10	120.8	90.5	423	45	55	45	680	627	484	607	382	+ 70
11	124.0	93.2	595	50	65	50	1110	830	1115	683	468	204
12	126.1	95.9	679	55	70	50	1226	893	1264	761	525	247
13	131.4	98.6	766	55	70	55	1347	955	1372	828	578	253
14	135.0	101.3	857	55	75	55	1440	1024	1476	890	672	307
15	138.9	104.0	987	60	80	60	1635	1141	1609	983	853	322
16	142.1	106.7	1135	65	85	65	1810	1248	1758	1080	979	403
17	146.0	109.4	1319	70	90	70	2475	1341	1970	1160	1117	427
18	149.2	112.1	1536	70	95	75	5890	1394	2720	1263	1301	517
19	150.1	114.8	1751	75	100	80	7650	1436	3893	1339	1557	556
20	152.0	114.8	2013	80	85	85	6442	1456	4940	1477	1880	706
21	152.0	114.8	2328	80	60	85	-	1506	3180	1559	2062	908
22	145.4	114.8	2678	80	25	100	-	1487	2278	1570	1808	1363

TABLE A.10 BEAM 124

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	12.2	0	0	0	0	0	0	0	0	0	0
1	36.4	12.2	14	5	0	0	3	5	3	7	-	1
2	44.5	14.9	26	0	0	5	5	7	10	8	+	1
3	52.6	17.6	36	5	0	5	6	13	14	20	5	5
4	60.7	20.3	43	0	5	10	8	15	24	21	7	7
5	68.7	23.0	54	5	0	5	13	22	35	34	8	8
6	77.0	25.7	67	5	5	5	18	24	46	46	15	15
7	85.1	28.4	77	5	5	10	23	34	63	57	21	21
8	93.4	31.1	95	5	5	10	26	32	89	73	34	34
9	101.5	33.8	115	0	5	10	31	41	131	90	49	49
10	109.5	36.5	141	0	5	10	28	49	245	122	74	74
11	117.8	39.2	172	-	5	10	17	125	386	137	115	115
12	125.9	41.9	459	-	5	5	-51	513	1378	397	405	405
13	130.0	43.3	566	-	5	5	-19	712	1518	513	556	556
14	134.0	44.6	659	-10	-10	5	+96	870	1648	624	734	734
15	138.0	46.0	786	-15	-15	0	670	1056	1814	732	983	983
16	142.0	47.3	924	-20	-25	-	920	1227	2045	870	1300	1300
17	146.0	48.7	1128	-20	-30	-10	1372	1442	3015	1059	1647	1647
18	150.2	50.0	1316	-25	-45	-20	1627	1548	5164	1161	1820	1820
19	152.1	51.4	1624	-30	-60	-20	1705	1641	6640	1300	1660	1660
20	150.8	51.4	1928	-35	-90	-35	1653	1597	7704	1338	603	603
21	138.2	51.4	2178	-30	-110	-40	1627	1679	7607	1408	488	488
22	-	51.4	-	-	-	-	-	-	-	-	-	-

TABLE A.11 BEAM 125

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. $\times 10^6$	DEFLECTIONS $\text{IN.} \times 10^3$			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
				EAST	CENTER	WEST	1	2	3	4	5	6
0	0	-	0	-	-	-	0	0	0	0	0	0
1	16.2	-	11	-	-	-	-1	3	0	6	-6	0
2	32.4	-	26	-	-	-	-2	-3	-1	0	-7	1
3	48.6	-	50	-	-	-	0	-2	+5	-2	-15	-5
4	56.7	-	61	-	-	-	3	-1	8	+4	-17	-4
5	64.8	-	73	-	-	-	6	+3	13	4	-22	-3
6	72.9	-	88	-	-	-	10	20	23	18	-22	-2
7	81.0	-	102	-	-	-	16	31	38	29	-28	+4
8	89.1	-	121	-	-	-	33	39	79	55	-31	16
9	90.0	-	153	-	-	-	43	56	154	58	-32	37
10	102.1	-	166	-	-	-	62	70	217	84	-33	55
11	106.8	-	194	-	-	-	67	87	308	103	-36	124
12	111.7	-	271	-	-	-	187	231	722	136	-52	372
13	115.1	-	425	-	-	-	276	776	1111	283	+71	651
14	118.2	-	552	-	-	-	551	932	1376	600	138	935
15	121.5	-	694	-	-	-	787	1080	1566	1338	282	1040
16	124.6	-	789	-	-	-	930	1178	1749	1473	360	1160
17	126.4	-	856	-	-	-	1018	1243	1900	1550	388	1214
18	128.0	-	925	-	-	-	1103	1300	2019	1620	429	1312
19	131.0	-	1026	-	-	-	1188	1370	2134	1641	455	1355
20	134.5	-	1132	-	-	-	1267	1408	2217	1688	521	1469
21	137.6	-	1235	-	-	-	1328	1447	2328	1736	576	1548
22	141.0	-	1361	-	-	-	1389	1468	2626	1827	651	1675
23	144.0	-	1536	-	-	-	1491	1508	3930	2116	746	1796
24	146.9	-	1770	-	-	-	1566	-	5730	2757	854	2020
25	147.5	-	2017	-	-	-	1636	-	6786	3145	919	2097
26	145.1	-	2293	-	-	-	1678	-	7064	3164	788	2001
27	140.1	-	2534	-	-	-	1694	-	7043	3104	562	1778

TABLE A.12 BEAM 126

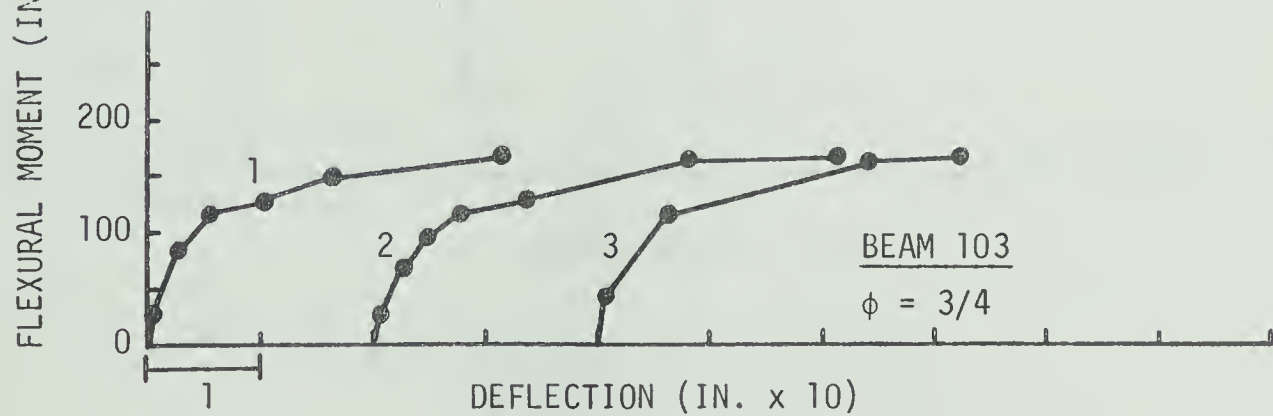
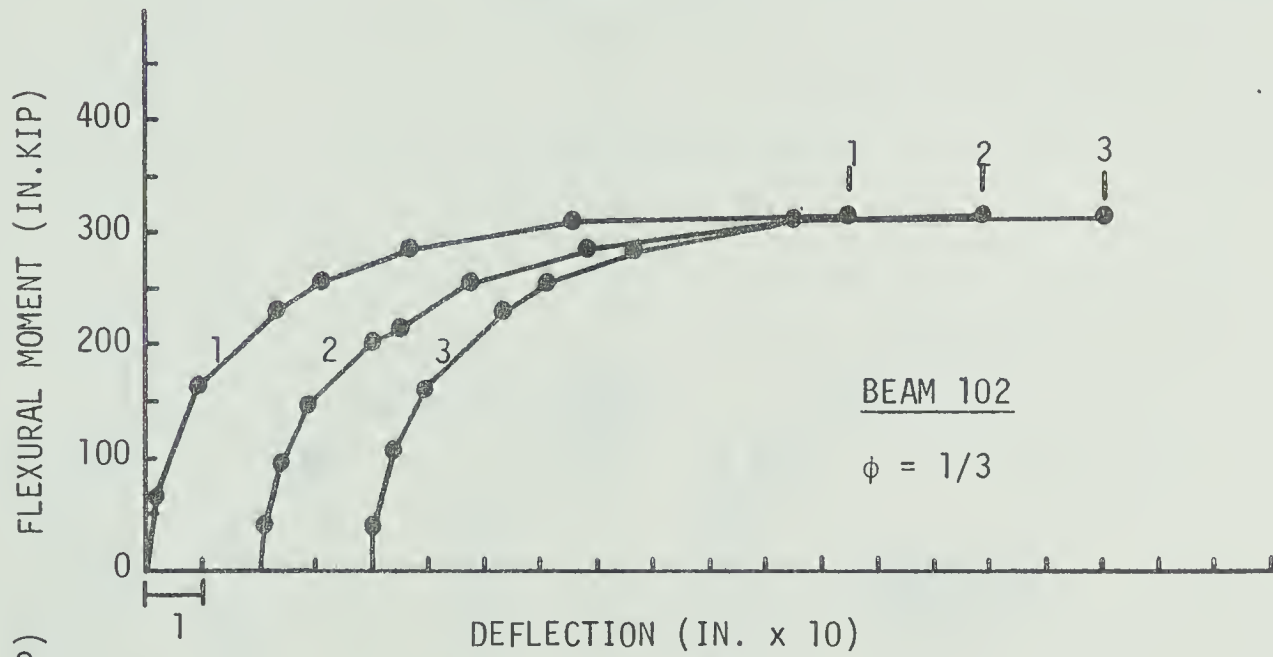
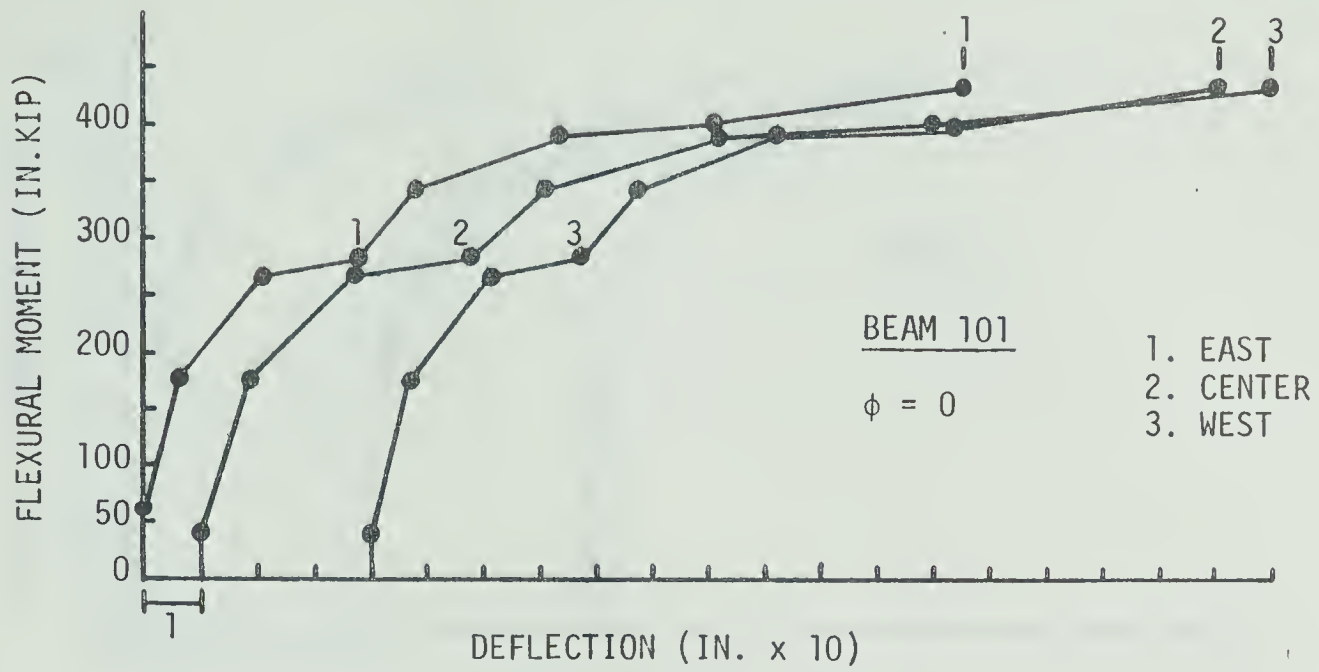


FIG. A.1 MOMENT-DEFLECTION CURVES
(BEAMS 101 to 103)

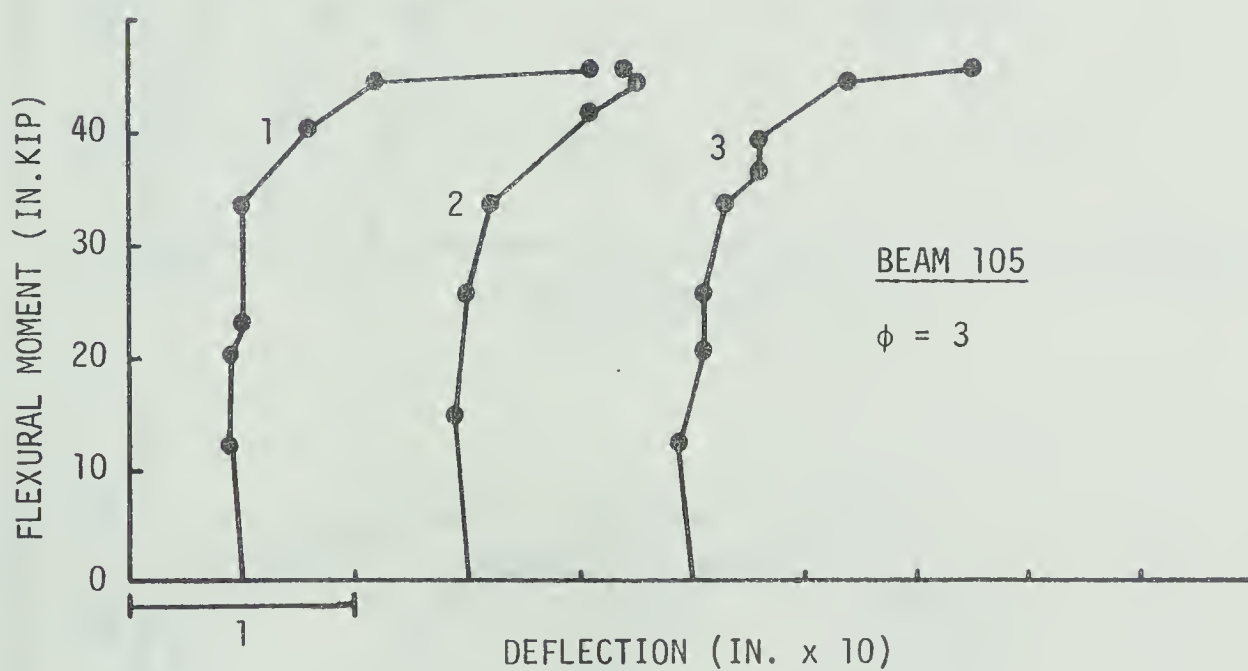
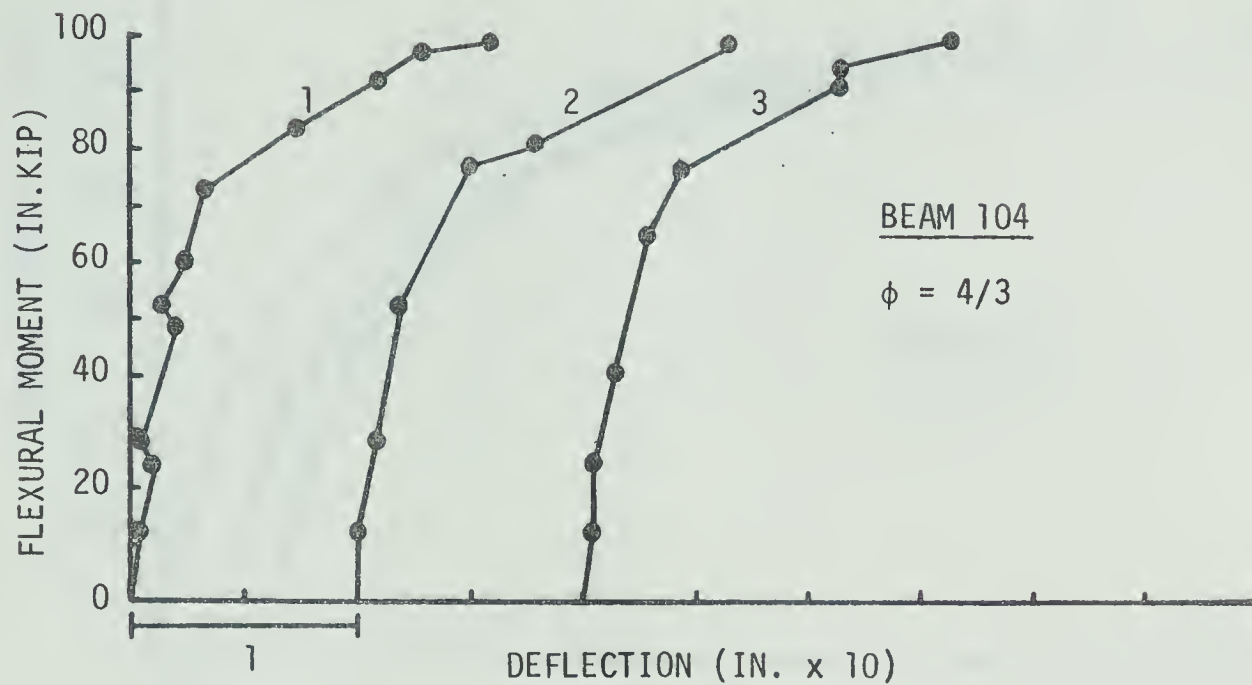


FIG. A.2 MOMENT-DEFLECTION CURVES
 (BEAMS 104 and 105)

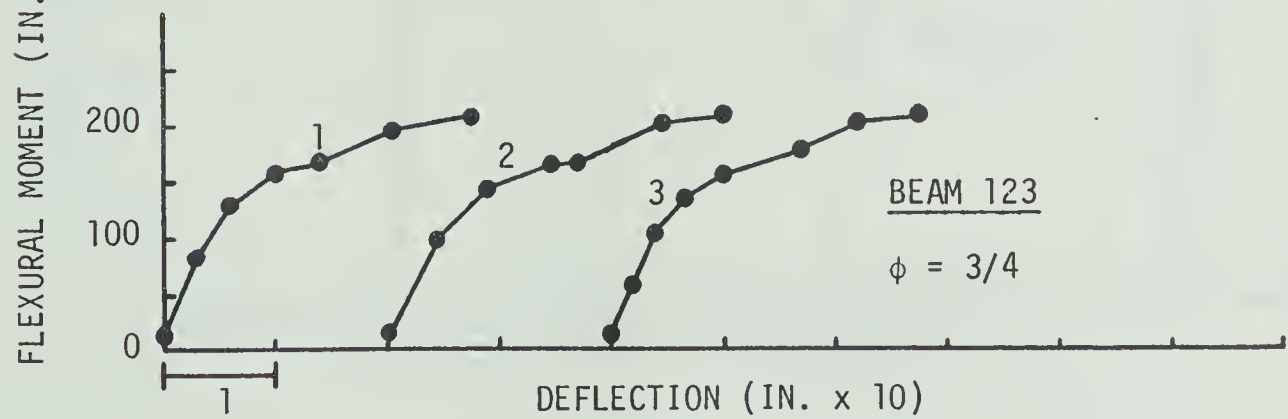
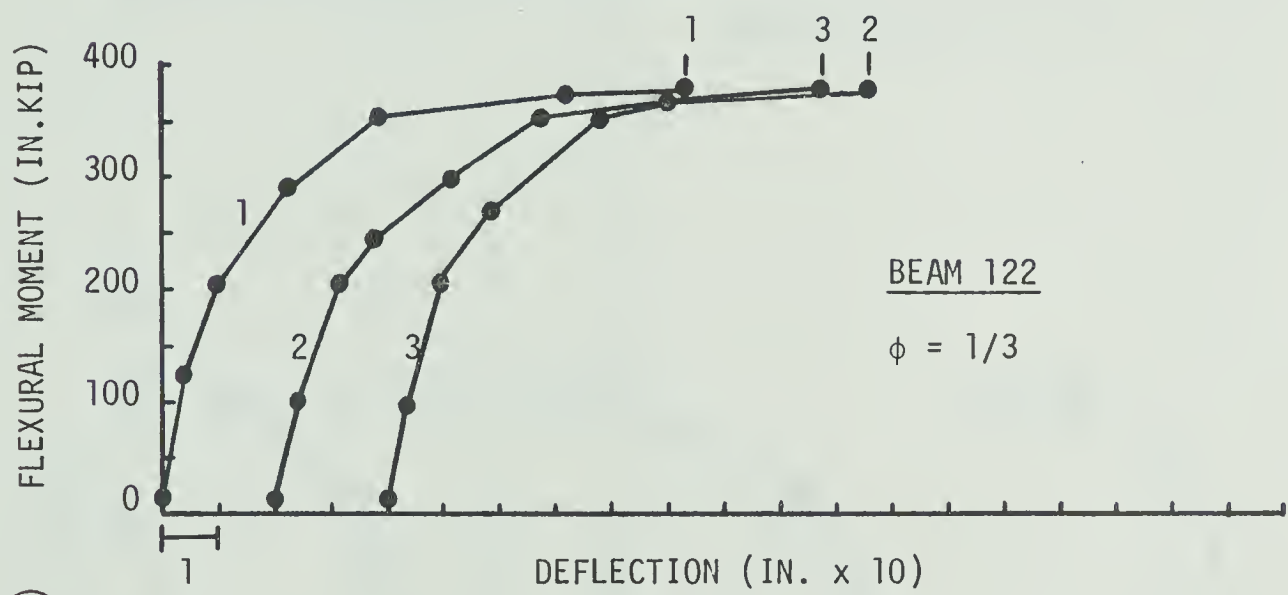
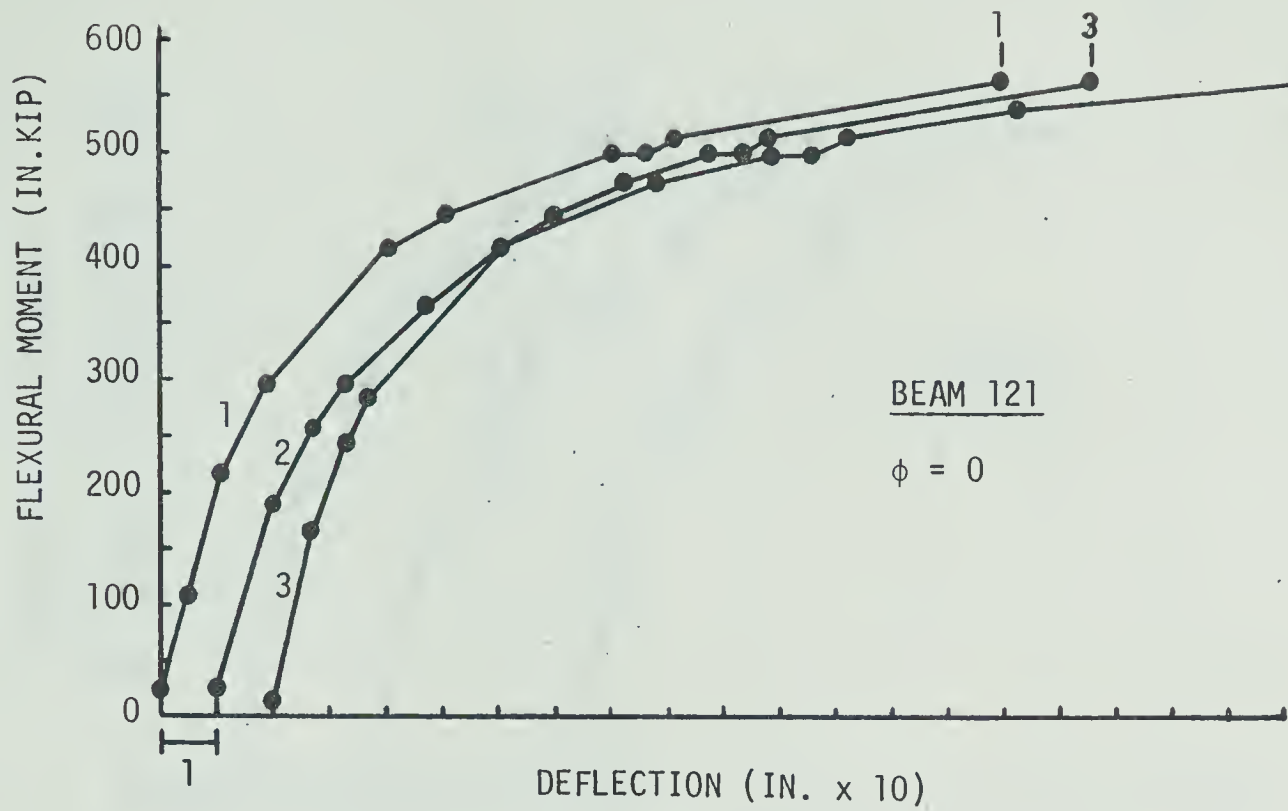


FIG. A.3 MOMENT-DEFLECTION CURVES
(BEAMS 121 to 123)

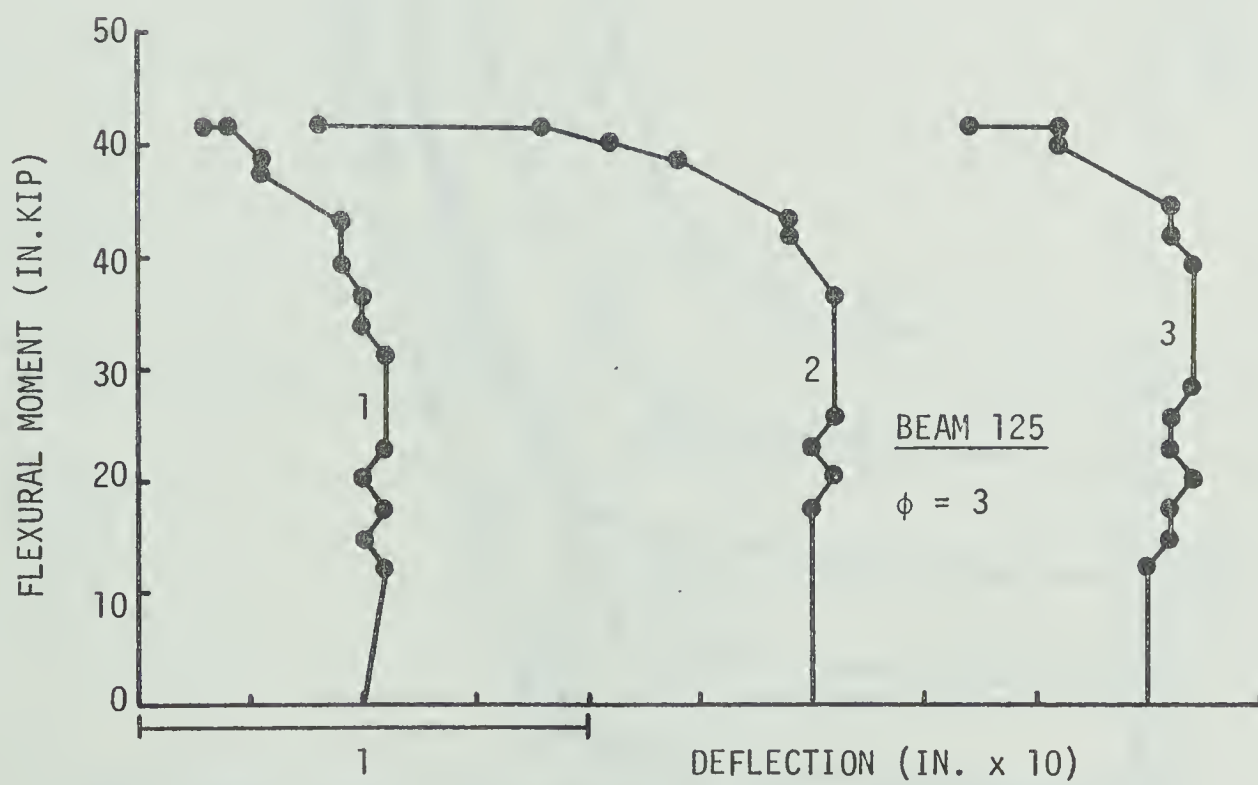
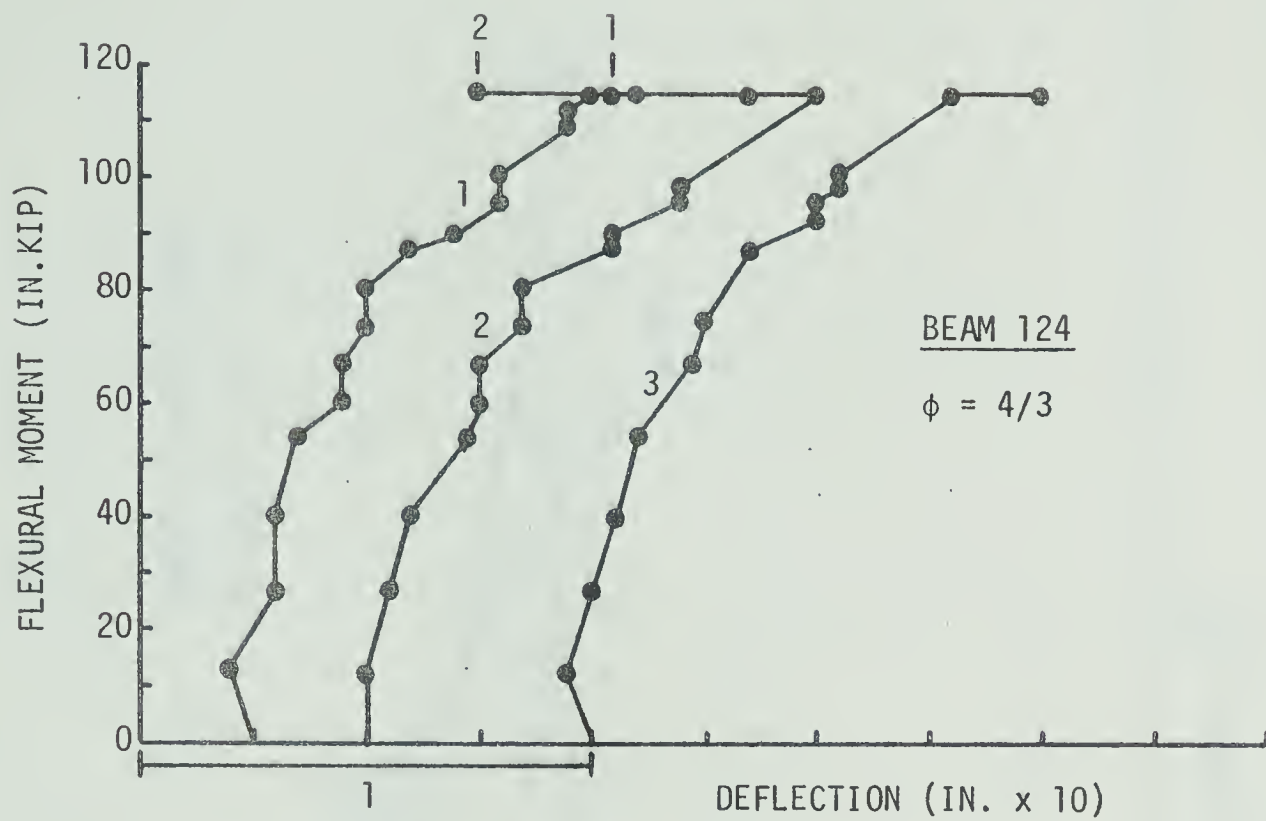


FIG. A.4 MOMENT-DEFLECTION CURVES
(BEAMS 124 and 125)

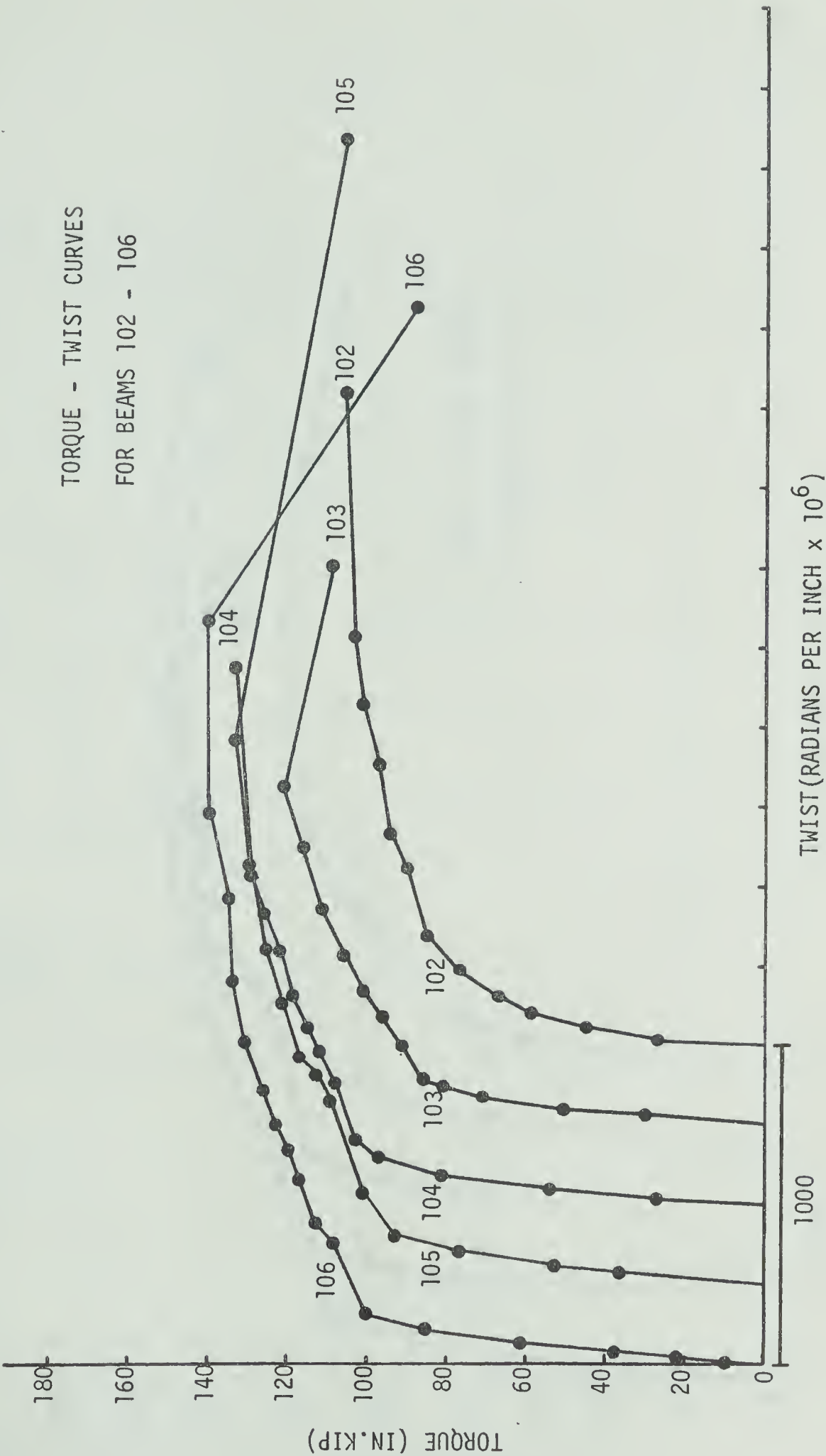


FIG. A.5 TORQUE-TWIST CURVES
(BEAMS 102 to 106)

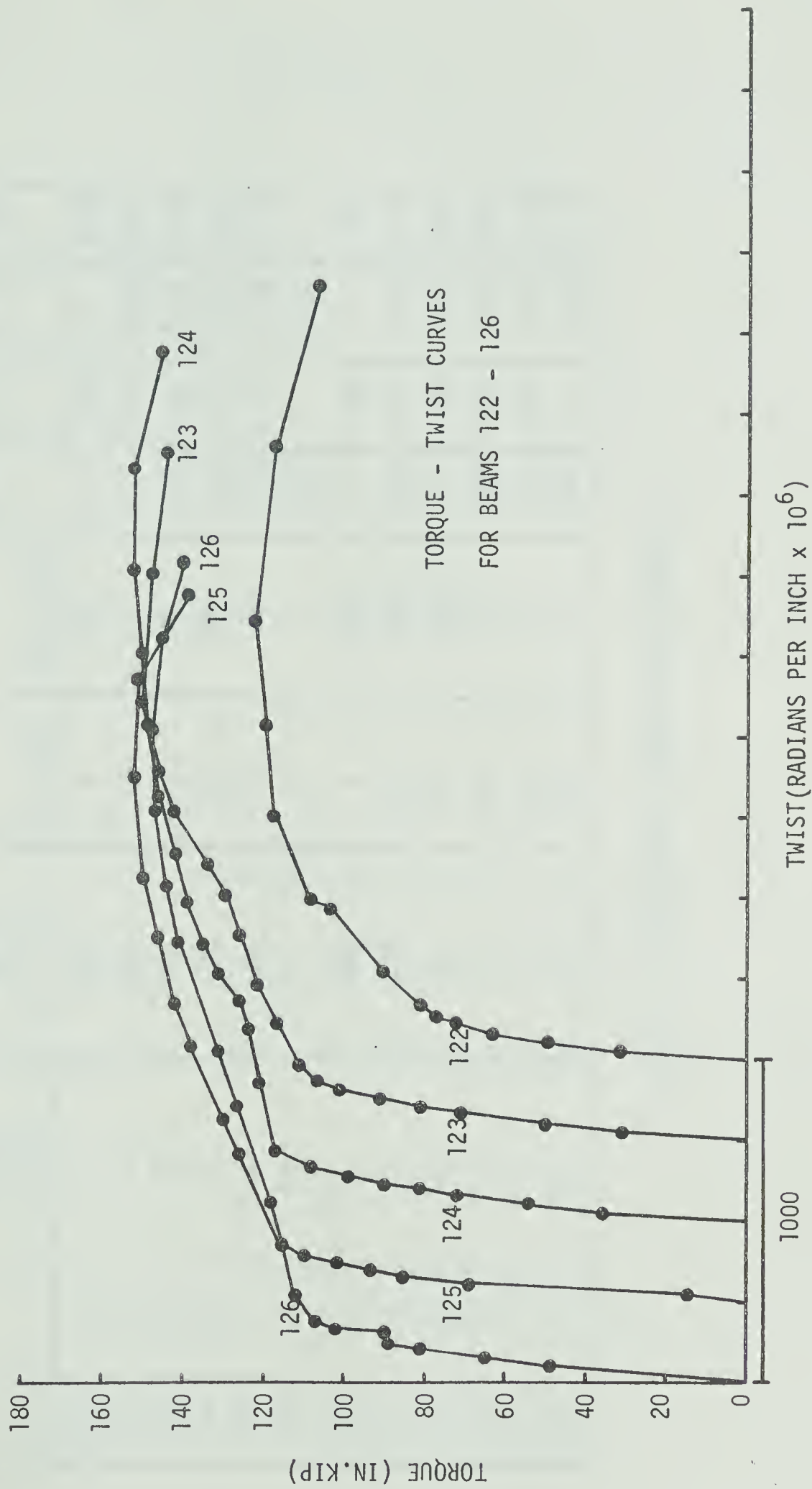


FIG. A.6 TORQUE-TWIST CURVES
(BEAMS 122 to 126)

BEAM NO.	ϕ RATIO	EXPERIMENTAL CRACKING TORQUE IN. KIPS	EXPERIMENTAL CRACKING MOMENT IN. KIPS	ULTIMATE TORQUE IN. KIPS	ULTIMATE MOMENT IN. KIPS	$\frac{T_u}{T_{uo}}$	$\frac{M_u}{M_{uo}}$	$\frac{T_{cr}}{T_{cro}}$	$\frac{M_{cr}}{M_{cro}}$
101	0	0	265	0	437	0	1.00	0	1.00
102	1/3	68	203	106	317	0.75	0.73	0.68	0.77
103	3/4	85	113	125	169	0.89	0.39	0.85	0.43
104	4/3	97	72	133	100	0.95	0.23	0.97	0.27
105	3	94	30	138	46	0.98	0.11	0.94	0.11
106	∞	100	0	141	0	1.00	0	1.00	0
121	0	0	340	0	567	0	1.00	0	1.00
122	1/3	81	245	122	365	0.83	0.64	0.76	0.72
123	3/4	107	142	150	200	1.02	0.35	1.00	0.42
124	4/3	117	87	152	115	1.03	0.20	1.09	0.26
125	3	115	38	152	51	1.03	0.09	1.07	0.11
126	∞	107	0	147	0	1.00	0	1.00	0

TABLE A.13 DATA FOR INTERACTION DIAGRAMS

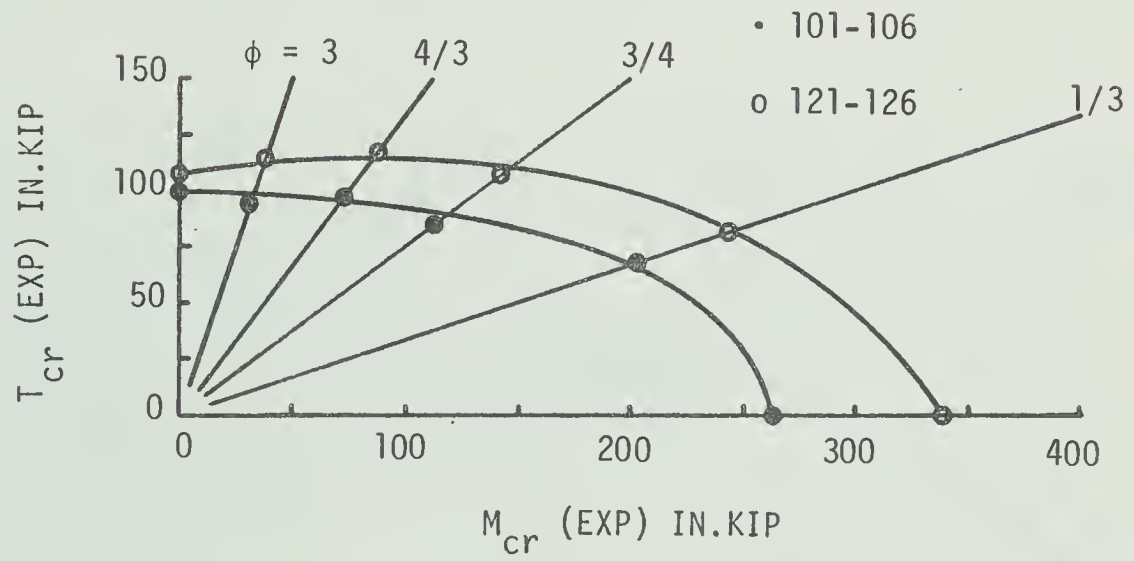
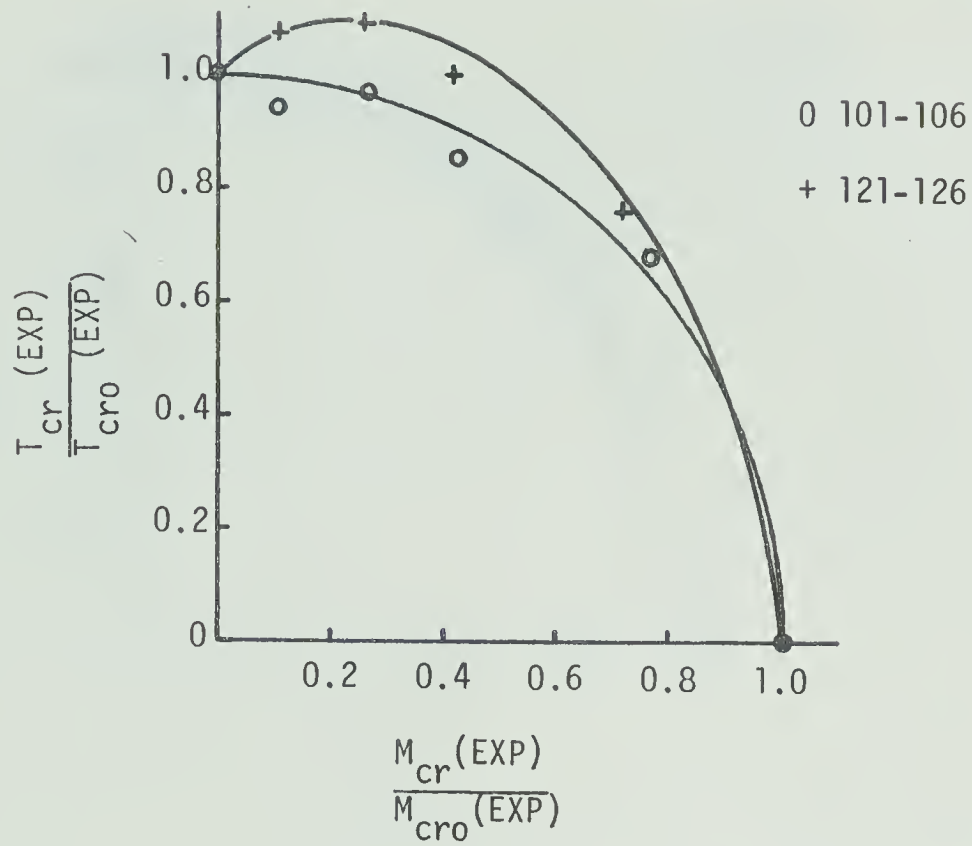


FIG. A.7 CRACKING LOAD
INTERACTION DIAGRAMS

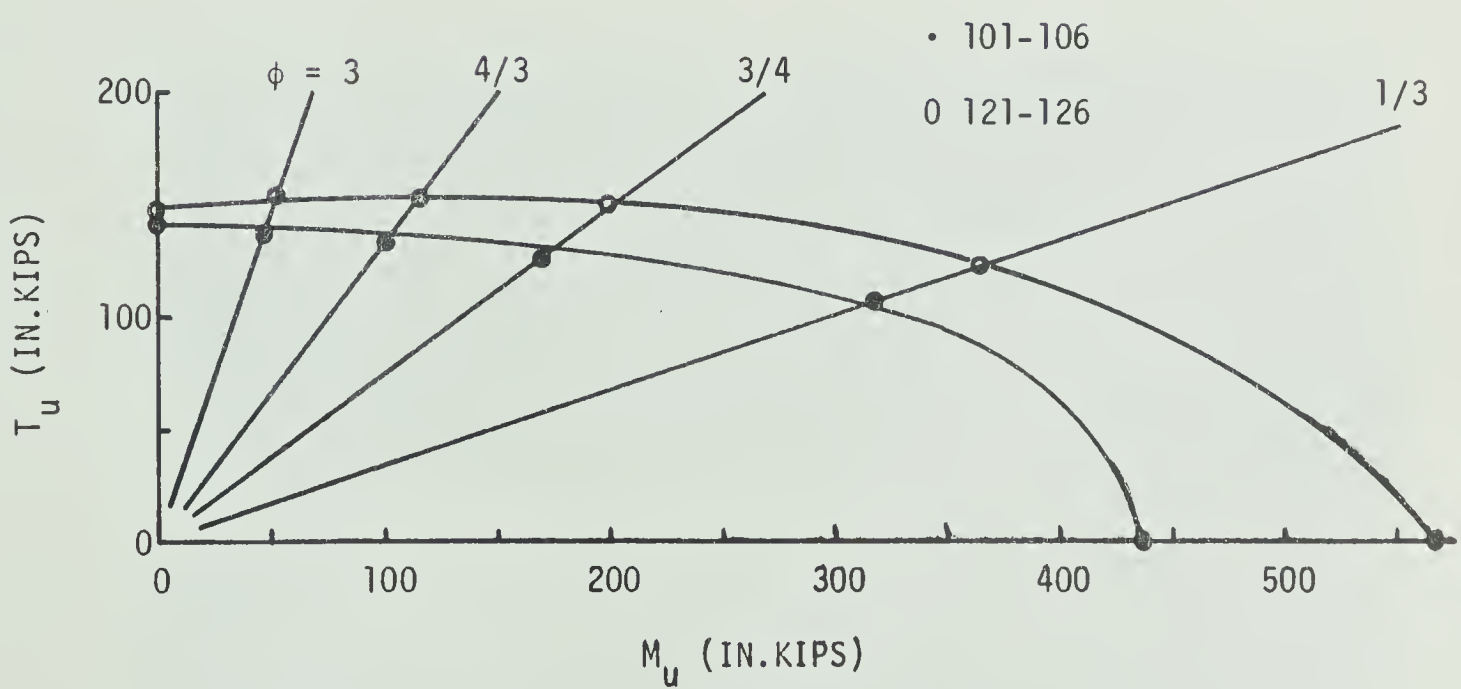
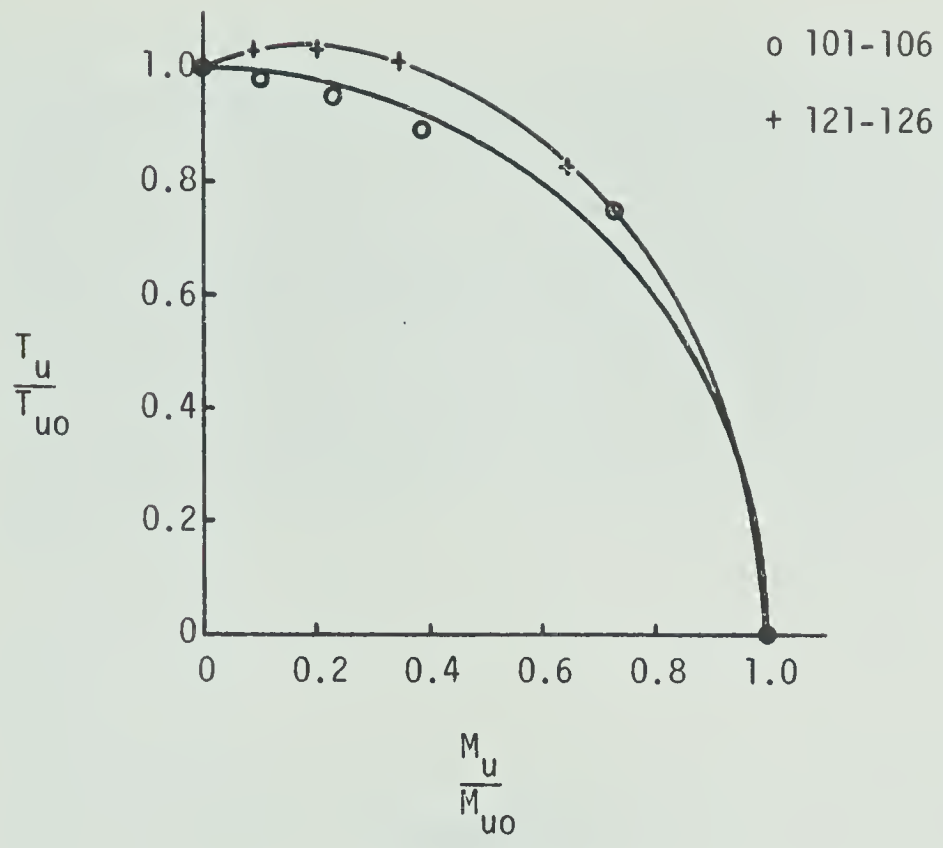
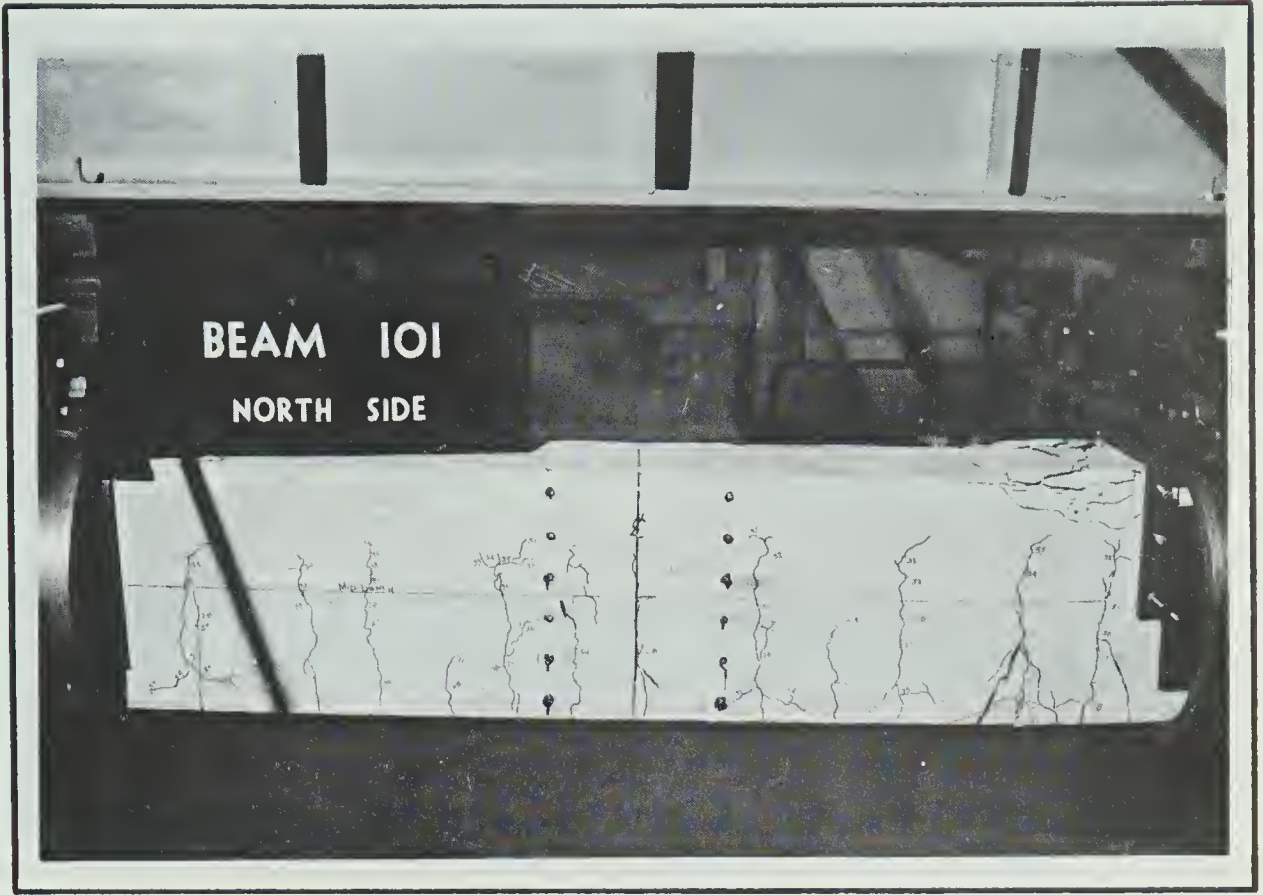


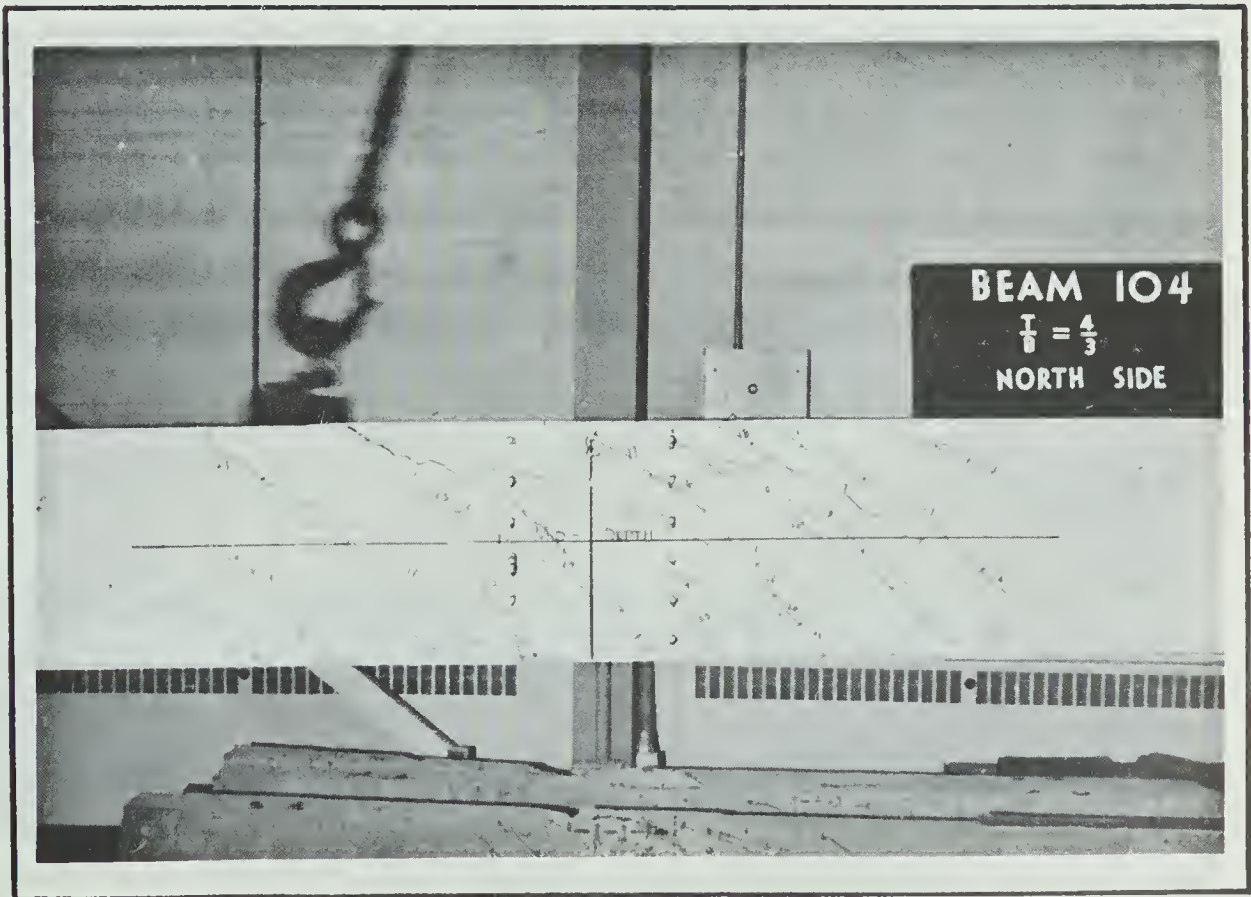
FIG. A.8 ULTIMATE LOAD
INTERACTION DIAGRAMS

APPENDIX B

PHOTOGRAPHS OF SPECIMENS

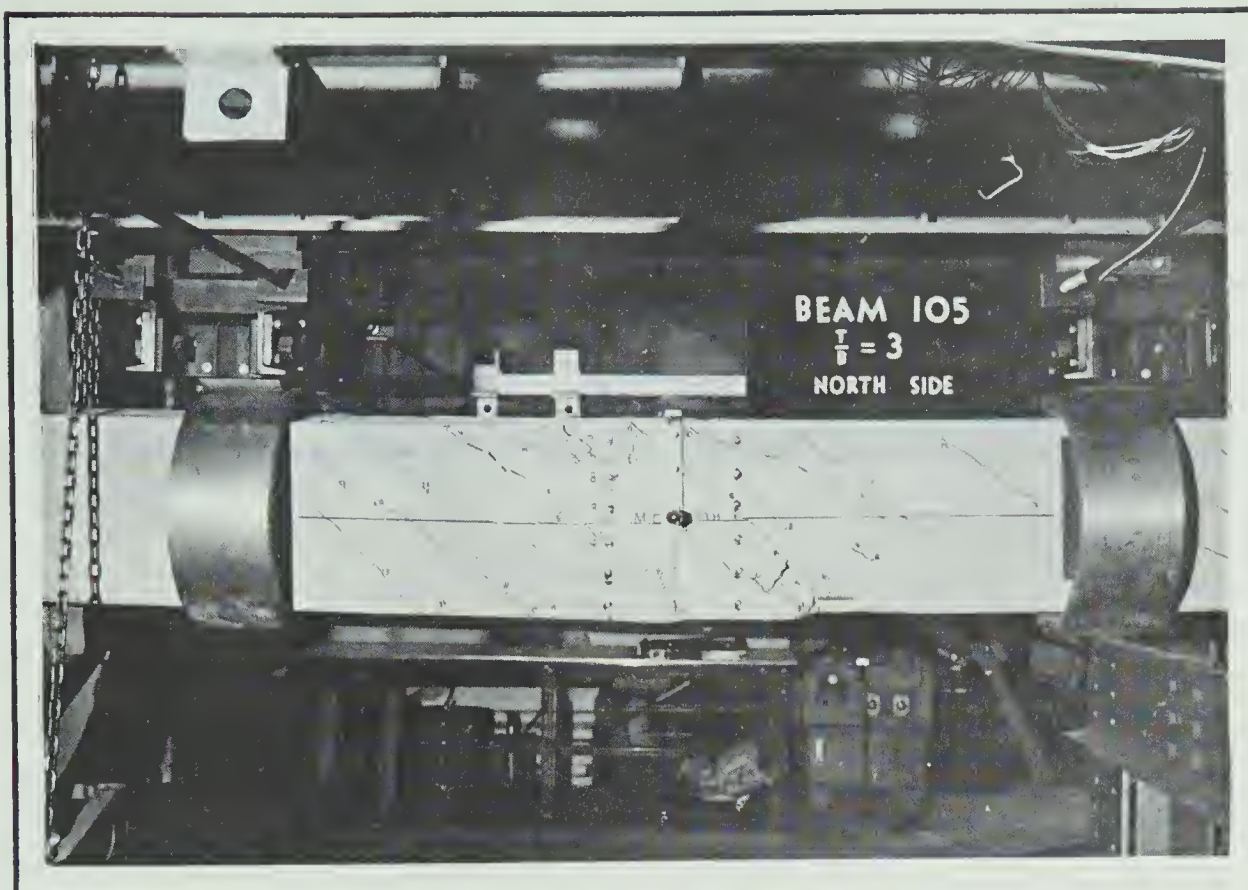


BEAM 101

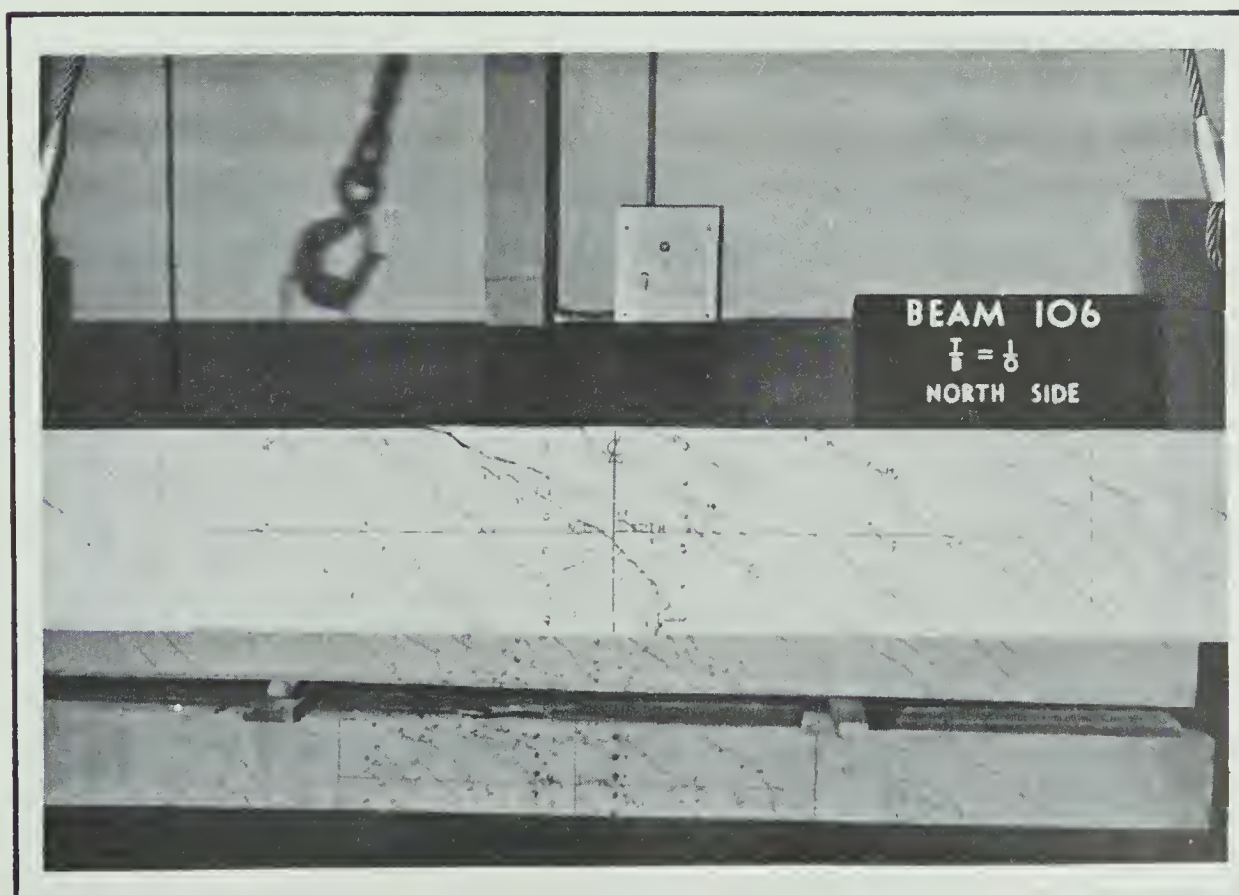


BEAM 104

FIGURE B.1 CRACK PATTERNS

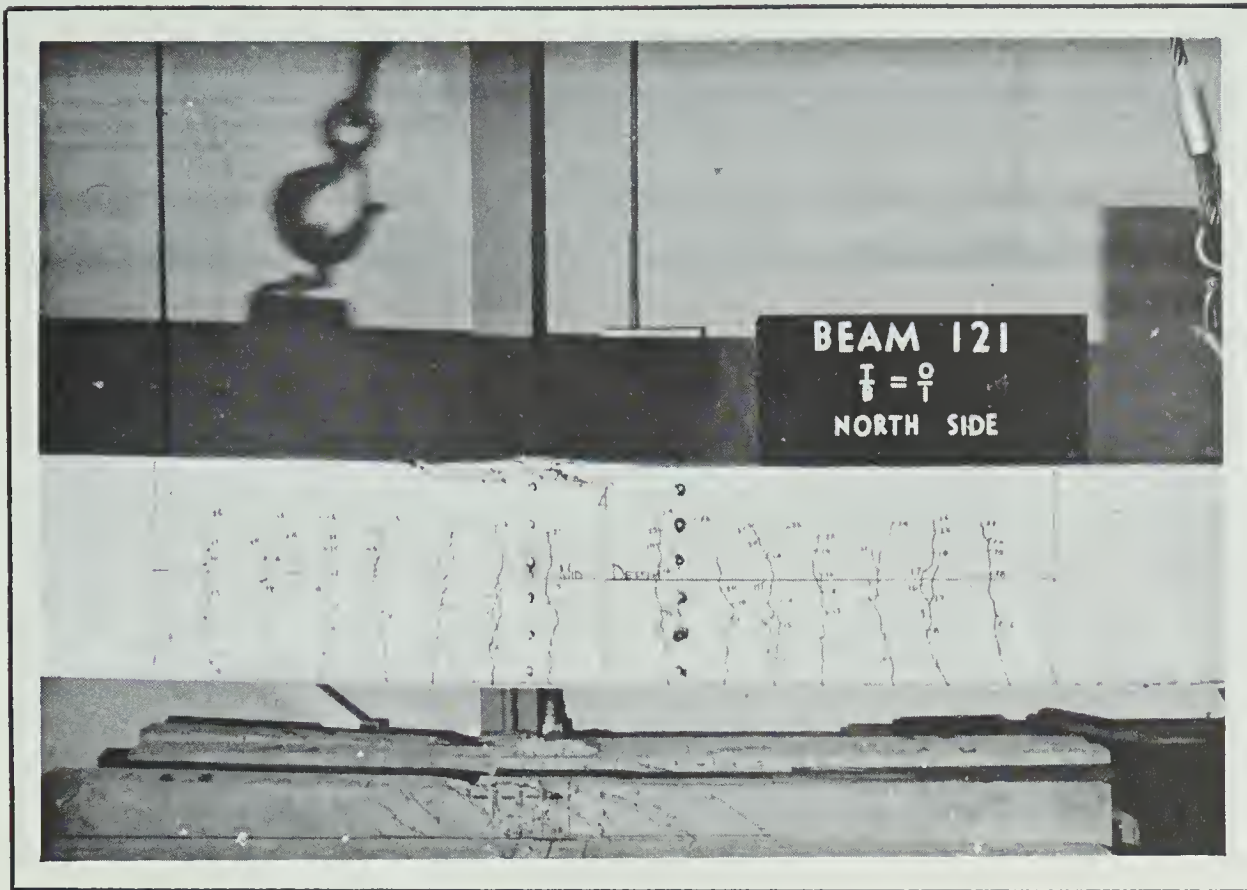


BEAM 105

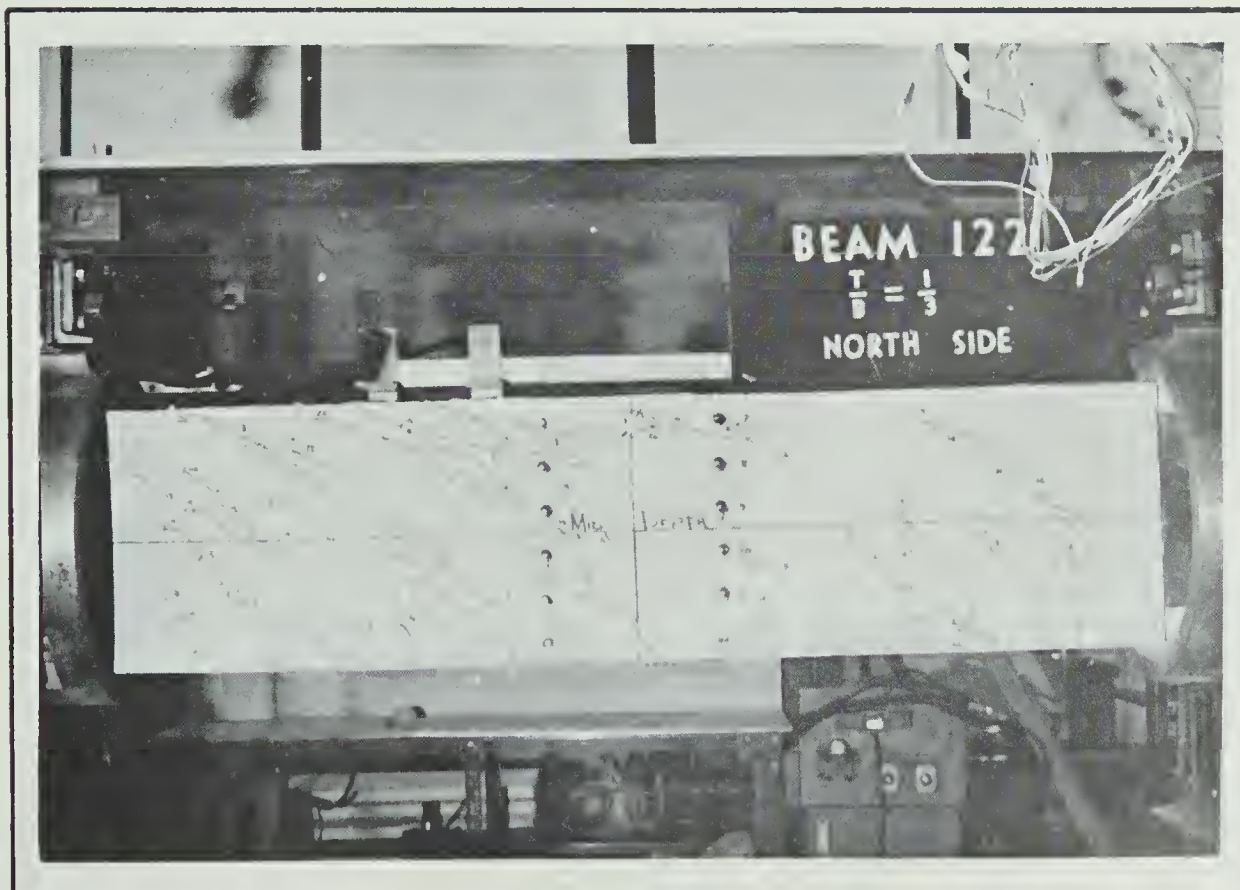


BEAM 106

FIGURE B.2 CRACK PATTERNS

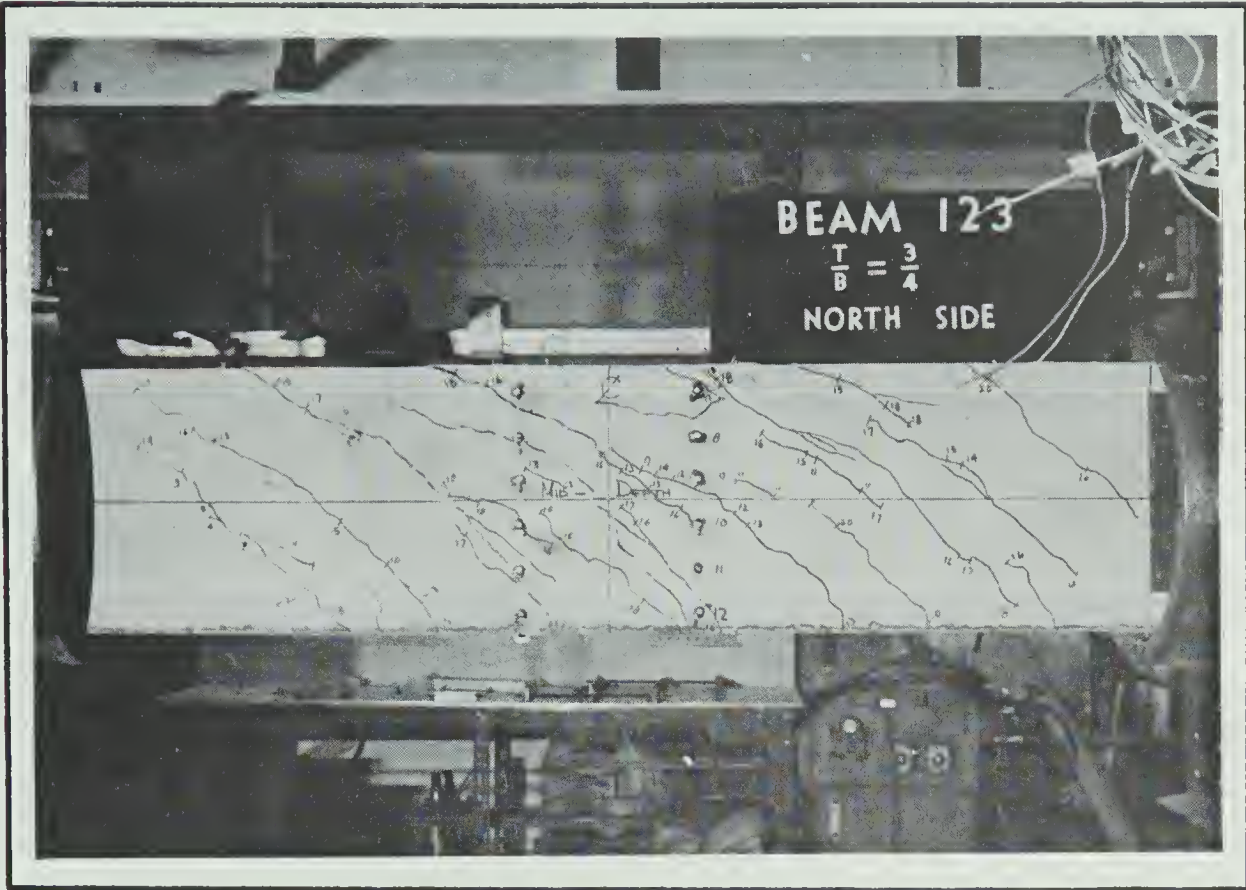


BEAM 121

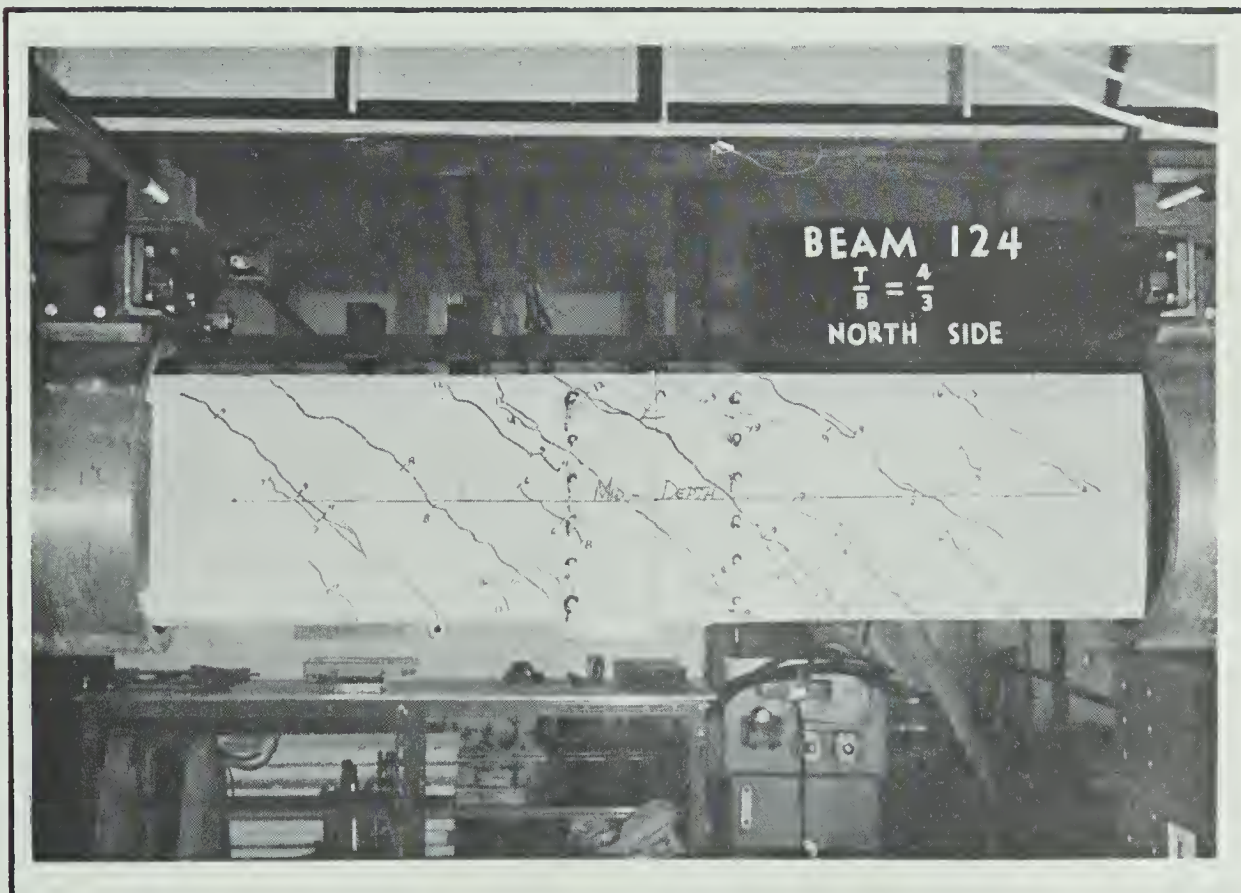


BEAM 122

FIGURE B.3 CRACK PATTERNS

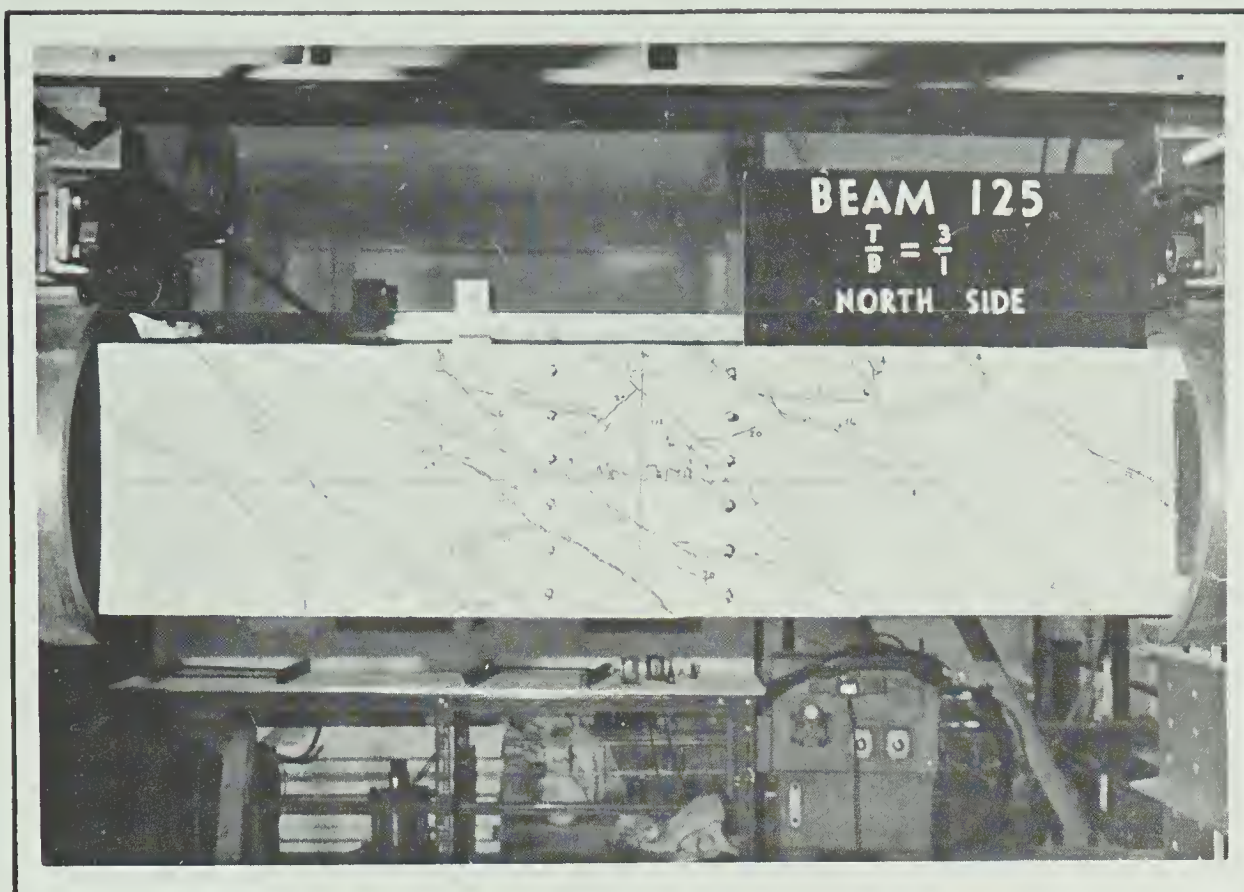


BEAM 123

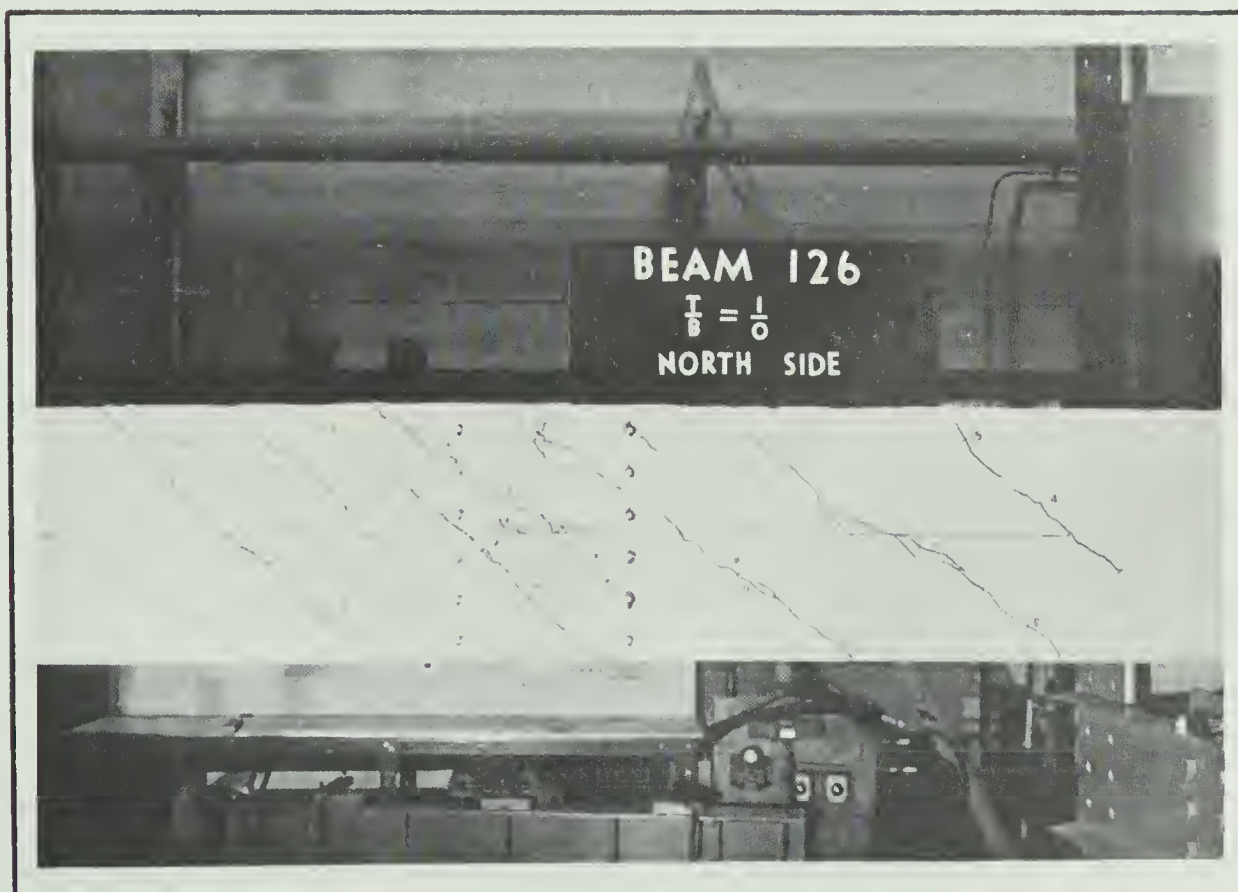


BEAM 124

FIGURE B.4 CRACK PATTERNS



BEAM 125



BEAM 126

FIGURE B.5 CRACK PATTERNS

APPENDIX C

NOTATION

NOTATION

f'_c	= compressive strength of concrete determined by tests on 6 x 12 inch concrete cylinders
f'_{sp}	= tensile strength of concrete as determined by tensile splitting tests
f_{yl}	= yield strength of longitudinal reinforcement
f_{yt}	= yield strength of lateral reinforcement
P_l	= longitudinal steel percentage
P_t	= transverse steel percentage
T_{cr}	= torsional cracking moment under combined load
T_{cro}	= torsional cracking moment under pure torsion
M_{cr}	= flexural cracking moment under combined load
M_{cro}	= flexural cracking moment under pure flexure
T_u	= ultimate torsional moment under combined load
T_{uo}	= ultimate torsional moment under pure torsion
M_u	= ultimate flexural moment under combined load
M_{uo}	= ultimate flexural moment under pure flexure
ϕ	= ratio of torsional moment to flexural moment

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